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UNIVALENCE OF SOLUTION OF THE EXTERIOR INVERSE BOUNDARY VALUE PROBLEM WITH RESPECT TO PARAMETERS x, y

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In paper [1] the solution of the exterior inverse boundary value problem (IBVP) with respect to parameters x, y was constructed. The objective of this article is to obtain sufficient conditions for univalence; however, to this end we should slightly change the statement of the problem in comparison with [1].

It is required to find on the plane of the complex variable $z = x+iy$ a contour $\partial D_z = L_z = L_z^1 \cup L_z^2$ and a function $w(z)$ analytic in the domain D_z, $\infty \in D_z$, via the boundary value condition

$$\begin{aligned} w &= \varphi_1(x) + i\psi_1(x), \quad 0 \le x \le l, \quad \text{on} \ \ L_z^1, \\ w &= \varphi_2(y) + i\psi_2(y), \quad 0 \le y \le l, \quad \text{on} \ \ L_z^2. \end{aligned} \tag{1}$$

Let us assume that the given univalent functions define on the plane w a closed Jordan contour L_w which is the boundary of a simply connected finite domain $D_w = w(D_z)$, $\partial D_w = L_w = L_w^1 \cup L_w^2$. We also assume that the point $w_0 = w(\infty)$ is a priori fixed (see [2], p. 17). After mapping of D_w onto the unit disk $E = \{|\zeta| < 1\}$, which sends w_0 to 0, while the points of the junction of arcs L_w^1 and L_w^2 to the points $e^{i\gamma_A}$ and $e^{i\gamma_B}$, respectively, and $0 < \gamma_A < \pi$, $\gamma_B = 2\pi - \gamma_A$, in order to find a function $z(\zeta)$ analytic in E with except for a pole in zero, we obtain the Hilbert boundary value problem

$$\begin{aligned} &\operatorname{Re} z(e^{i\gamma}) = x(\gamma), \quad \gamma_A \le \gamma \le \gamma_B, \\ &\operatorname{Im} z(e^{i\gamma}) = y(\gamma), \quad 0 \le \gamma \le \gamma_A, \quad \gamma_B \le \gamma \le 2\pi, \end{aligned} \tag{2}$$

where $x(\gamma)$ and $y(\gamma)$ are known monotone functions which are assumed to be Hölder. We denote by K the class of all functions φ_1, ψ_1, φ_2, ψ_2, which satisfy all the above conditions. The two cases can take place here: $x(\gamma)$ decreases while $y(\gamma)$ increases, or vice versa. For the sake of definiteness, we assume that the first possibility is realized.

In a way analogous to that in [1] we define a solution of problem (2) in the form

$$z(\zeta) = -iF_0(\zeta)\Big(\frac{1}{2\pi}\int_0^{2\pi} \frac{c(\sigma)}{|F_0(e^{i\sigma})|}\,\frac{e^{i\sigma}+\zeta}{e^{i\sigma}-\zeta}d\sigma + iB_0 + C\zeta - \frac{\overline{C}}{\zeta}\Big), \tag{3}$$

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