Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 49, No. 8, pp.53–58, 2005 *UDC 517.956*

THE GENERAL CASE OF THE GOURSAT PROBLEM

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Let $D=\{x_{10}<x_1<x_{11},\ x_{20}<x_2<x_{21},\dots,x_{n0}<x_n<x_{n1}\}$. Denote by $X_1,X_2,\dots,X_n$ the facets of D for $x_1=x_{10}$, $x_2=x_{20},\dots,x_n=x_{n0}$, correspondingly.

Consider the equation

$$L(u)\equiv\sum_{i_1=0}^{m_1}\sum_{i_2=0}^{m_2}\cdots\sum_{i_n=0}^{m_n}a_{i_1i_2\dots i_n}(x_1,x_2,\dots,x_n)\frac{\partial^{i_1+i_2+\dots+i_n}u}{\partial x_1^{i_1}\partial x_2^{i_2}\dots\partial x_n^{i_n}}=F(x_1,x_2,\dots,x_n). \tag{1}$$

We study it in D; here $a_{m_1m_2\dots m_n}\equiv 1$ and the smoothness of other coefficients is defined by the inclusions

$$a_{i_1i_2\dots i_n}\in C^{\sum\limits_{\alpha=1}^{n}i_\alpha}(\overline{D}),\quad F\in C^{0+0+\dots+0}(\overline{D}).$$

Here $C^{\alpha_1+\alpha_2+\dots+\alpha_n}$ is the class of continuous in $\overline{D}$ functions with continuous derivatives $\frac{\partial^{r_1+r_2+\dots+r_n}}{\partial x_1^{r_1}\partial x_2^{r_2}\dots\partial x_n^{r_n}}$ $(r_1=0,\dots,\alpha_1;\ r_2=0,\dots,\alpha_2;\dots;r_n=0,\dots,\alpha_n)$. Equation (1) is the most general linear equation of this class.

This equation is studied in [1] on a plane. Its special cases with $n>2$ variables are considered in [2], [3], and other works, including those of the author [4]–[6].

The Goursat problem. Find in D a regular solution of equation (1) which satisfies the conditions

$$\begin{aligned}
&\frac{\partial^{i_1}u}{\partial x_1^{i_1}}(x_{10},x_2,\dots,x_n)=\varphi_{1\,i_1}(x_2,\dots,x_n)\quad (i_1=\overline{0,m_1-1}),\\
&\frac{\partial^{i_2}u}{\partial x_2^{i_2}}(x_1,x_{20},\dots,x_n)=\varphi_{2\,i_2}(x_1,x_3,\dots,x_n)\quad (i_2=\overline{0,m_2-1}),\\
&\qquad\qquad\dots\dots\dots\dots\\
&\frac{\partial^{i_n}u}{\partial x_n^{i_n}}(x_1,x_2,\dots,x_{n0})=\varphi_{n\,i_n}(x_1,\dots,x_{n-1})\quad (i_n=\overline{0,m_n-1}),
\end{aligned} \tag{2}$$

$$\varphi_{1\,i_1}\in C^{\sum\limits_{\alpha=2}^{n}m_\alpha}(\overline{X}_1),\ \varphi_{2\,i_2}\in C^{\sum\limits_{\alpha=1,\,\alpha\neq 2}^{n}m_\alpha}(\overline{X}_2),\dots,\varphi_{n\,i_n}\in C^{\sum\limits_{\alpha=1}^{n-1}m_\alpha}(\overline{X}_n),$$

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