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Asymptotic Stability of Solutions of a Class of Systems of Nonlinear Differential Equations with Delay

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Abstract—We study systems of differential equations with delay whose right-hand sides are represented as sums of potential and gyroscopic components of vector fields. We assume that in the absence of a delay zero solutions of considered systems are asymptotically stable. By the Lyapunov direct method, using the Razumikhin approach, we prove that in the case of essentially nonlinear equations the asymptotic stability of zero solutions is preserved for any value of the delay.

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1. INTRODUCTION

Systems of differential equations with a delayed argument are widely used for modeling various real-life phenomena and processes [1–3]. One of the most actual problems, which arise in the analysis of the dynamics of such systems, is the stability problem [2–4]. Studying the stability of solutions, one should take into account the influence of the delay on it. It is known [1, 2] that even small delay values can lead to the loss of stability. Therefore the great interest is presented by the problem of finding the limit delay values which do not violate the stability of solutions, including the description of classes of systems whose solutions remain stable with any delay values.

The main method of the stability analysis in nonlinear systems is the direct Lyapunov method. When applying it to studying systems with a delayed argument, one uses either the Lyapunov–Krasovskii functionals [2–5] or the Lyapunov functions and the B. S. Razumikhin approach [1–4, 6]. With the help of the indicated approaches one has obtained stability conditions for solutions of many types of systems with delay [1–6]. However, by now there exist no general constructive techniques for obtaining the Lyapunov functions or functionals for nonlinear systems.

In this paper we consider systems of nonlinear differential equations with a delayed argument whose right-hand sides are representable as sums of potential and gyroscopic components of vector fields. We assume that with the absence of delay, zero solutions of systems under consideration are asymptotically stable. Using the method of Lyapunov functions and the B. S. Razumikhin approach, we show that if the considered equations are essentially nonlinear, then the asymptotic stability takes place with any value of the delay. We estimate the duration of transient processes for asymptotically stable systems with delay. We study the stability of perturbed systems. We define types of nonlinear nonstationary perturbations, for which the zero solution remains asymptotically stable even if their order is less than the order of homogeneity of functions in the right-hand sides of unperturbed equations.

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