

Continuity of Ring Homomorphisms for Local C^* -Algebras

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Abstract—We show that a ring homomorphism from a local σ - C^* -algebra to a local C^* -algebra is a continuous mapping.

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INTRODUCTION

At present time, the theory of operator algebras is a highly developed branch of mathematics that lies at the intersection of algebra and functional analysis. The introduced by I. M. Gelfand and M. A. Naimark C^* -algebras represent the most investigated class of such algebras. Moreover, in the second half of the previous century in several papers one has studied the structure of more general involutory topological algebras, where the topology was defined by a family of seminorms satisfying C^* -axioms [1–4]. Now the structure theory of such algebras and modules over them is intensively developing (e.g., monographs [5–7] and papers [8–16]). This paper is also devoted to this subject. We prove that any ring \star -homomorphism acting from a local σ - C^* -algebra to a local C^* -algebra is a continuous mapping.

1. PRELIMINARY INFORMATION

All necessary information about C^* -algebras and more general topological algebras with involution can be found in [17, 5, 7]. The theory of matrix operator algebras is described in [18]. All algebras are considered over the field of complex numbers.

1.1. An algebra A with an involution is called an involutory LMC-*algebra*, if A is a local convex topological vector space, where the topology is defined by a family of seminorms $(P_\lambda)_{\lambda \in \Lambda}$ such that

1) $P_\lambda(xy) \leq P_\lambda(x)P_\lambda(y)$ for any $x, y \in A$ and any $\lambda \in \Lambda$,

2) $P_\lambda(x) = P_\lambda(x^*)$ for any $x \in A$ and $\lambda \in \Lambda$.

An involutory LMC-*algebra* is called a local C^* -*algebra*, if it is complete and the following condition is fulfilled:

3) $P_\lambda(xx^*) = P_\lambda(x)^2$ for any $x \in A$ and $\lambda \in \Lambda$.

Seminorms with the mentioned properties are called C^* -*seminorms*. If a local C^* -*algebra* has a unit element, then the algebra is said to be *unital*. Let A be a local C^* -*algebra*. If a family of C^* -*seminorms* in A is countable, then A is called a *local σ - C^* -algebra*.

1.2. Let us consider some examples.

Example 1. Each C^* -*algebra* is a local C^* -*algebra*.

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