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Uniqueness of a Positive Solution to the Dirichlet Problem for a Quasilinear Equation with p-Laplacian in a Ball

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Abstract—We obtain sufficient conditions for the existence of a unique positive radially symmetric solution to the Dirichlet problem for a quasilinear equation of elliptic type in a multidimensional ball.

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In the ball $S = \{x \in R^n : |x| \ < 1\}$ with boundary Γ we consider the Dirichlet problem

$$\Delta_p u + |x|^m |u|^q = 0, \ \ x \in S, \tag{1}$$

$$u_\Gamma = 0, \tag{2}$$

where $\Delta_p u = \mathrm{div}(|\nabla u|^{p-2}\nabla u)$, while $1 < p \leq 2$, $m \geq 0$, and $q > 1$ are constants.

We understand a positive solution to problem (1), (2) as its solution in the class $C^2(\overline{S})$ which is positive in S and satisfies the boundary condition (2).

Many works of Russian and foreign mathematicians are devoted to positive solutions of problems in the form (1), (2) (e.g., [1–9]). The unique existence of a positive radially symmetric (p. r. s.) solution to problem (1), (2) with $p = n = 2$ is proved in [8] for all $m \geq 0$ and $q > 1$. For $n \geq 3$ and $p = 2$ the unique existence of a p. r. s. solution to problem (1), (2) is proved in [9] under the assumption that $q \leq \frac{m+n}{n-2}$. In [7] one has proved the existence and non-existence theorems for p. r. s. solutions to the Dirichlet problem for Eq. (1) with a gradient term, namely, for the equation

$$\Delta_p u + \lambda |u|^q = |\nabla u|^s, \ \ q > 1, \ \ s \geq p - 1, \ \ \lambda > 0.$$

In this paper we study the unique existence of a positive solution to the Dirichlet problem for Eq. (1) with $n \geq 2$ and for the equation

$$\Delta_p u + \lambda r^m |u|^q = |\nabla u|^s, \ \ q > 1, \ \ s \geq p - 1, \ \ m \geq 0, \ \ \lambda > 0, \ \ 0 < r < 1, \tag{3}$$

with $n = 1$. We extend the results obtained in [8, 9] for p. r. s. solutions to quasilinear equations with p-Laplacian. We also continue the research initiated in [7] for $n = 1$; namely, we obtain sufficient conditions for the unique existence of a p. r. s. solution to Eq. (3).

1. The case $n \geq 2$. If there exists a radially symmetric solution to problem (1), (2), then it satisfies the following two-point boundary-value problem:

$$|u'|^{p-2}\left((p-1)u'' + \frac{n-1}{r}u'\right) + r^m |u|^q = 0, \ \ 0 < r < 1, \tag{4}$$

$$u'(0) = 0, \ \ u(1) = 0. \tag{5}$$

Using the Ts. Na transformation [10]

$$r = A^\alpha \overline{r}, \ \ u = A^\beta \overline{u}, \tag{6}$$

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