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FOURTH ORDER DIFFERENTIAL SYSTEMS WITH FOUR-DIMENSIONAL SOLVABLE GROUP OF SYMMETRIES WHICH DOES NOT CONTAIN AN ABELIAN SUBGROUP G_3

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1.

Tending to expand the Galois theory methods of solving algebraic equations by radicals to the problem of integrating differential equations, S. Lie laid the foundations for the theory of continuous transformation groups. The profound ideas of Lie related to application of Lie groups to differential equations were developed in [1]–[9] and other papers.

Lie tended to give a clear geometric interpretation to symmetries of differential equations. E. Cartan sought for a generalization of a metric space which would give a possibility to consider the integral curves of a system of second order ordinary differential equations as geodesics of a generalized space. Developing his theory of projectively connected spaces, Cartan emphasized its significance for the study of differential equations. Differential-geometric methods and, in particular, the methods of Cartan's theory make it possible to develop a systematic geometric approach to the determination and study of local and nonlocal symmetries of large classes of ordinary and partial differential equations and to finding solutions of differential equations under study.

The foundations of such an approach were developed in [10], [11] and other papers devoted to the projective geometry of systems of differential equations. In [10], we discussed group properties of equations of geodesics in Riemannian and affinely connected spaces. Every point symmetry of equations of geodesics in such a space is a projective transformation. On the other hand, projective transformations of Riemannian manifolds determine symmetries of Hamiltonian systems and Lie–Bäcklund transformations of Hamilton–Jacobi equations with quadratic Hamiltonians.

In previous papers, we studied group properties of second order systems of differential equations (resolved with respect to the highest derivatives) whose right-hand sides are cubic with respect to the first order derivatives. We make no preliminary assumptions concerning presence of a geometric structure (Riemannian, affine, and the like) in the space of dependent and independent variables of the system. We have established the law of transformation of the system for the general change of variables and proved that certain combinations of the coefficients of the system are transformed, under such a change, as the components of a projective connection. It is remarkable that every projective connection written in coordinates can be obtained in this way: any projective connection on an n-dimensional manifold M is determined locally by a system $\mathcal{S}$ of $n-1$ second order ordinary differential equations (resolved with respect to the second derivatives) whose right-hand sides are polynomials of degree three with respect to the derivatives of the unknown functions, and every differential system $\mathcal{S}$ determines an (associated) projective connection on M. In other words, the theory of systems $\mathcal{S}$ of differential equations is the theory of projective connections. It was proved that the group of symmetries of a differential system $\mathcal{S}$ is a group of projective transformations in an n-dimensional space with associated projective connection. This fact allows ones to use

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