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THE METHOD OF RIEMANN SURFACES FOR THE PROBLEM OF INTERFACIAL CRACKS AND INCLUSIONS IN THE PRESENCE OF POINT FORCES

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L.I. Chibrikova was among the first researchers who paid attention to the possibilities of using the Riemann surfaces to solve the problems of the elasticity theory. In [1], by reduction to the Riemann boundary-value problem on the Riemann surface, studied are the main problems of the elasticity theory for domains bounded by algebraic curves. In [2], the method of Riemann surfaces is applied for solving some plane boundary-value problems of the theory of analytic functions and singular integral equations. A lot of applied problems can be reduced to the mentioned ones. In [3] and [4], the stated method is used for solving the main problems of the elasticity theory for a plane with a finite number of collinear cuts, including the mixed problem with an arbitrary location of the points where the type of boundary conditions at banks of cuts changes. This problem is most complicated and most interesting for applications. Various cases of this problem are investigated using the same method in [5], [6]. In [7], described is the method which reduces a number of contact problems of the elasticity theory for a system of half-planes to the Riemann boundary-value problem on the Riemann surface. In [8]–[11], the Riemann surface is used for solving the main problems of the elasticity theory directly on the Riemann surfaces with cuts, as well as for the investigation of various models of an elastic helical surface. In all works mentioned above, considered are the problems for homogeneous elastic media. In the case of heterogeneous media and structures, the method of Riemann surfaces is applied to the solution of various problems of the elasticity theory [12]–[14]. In [15] and [16], described are the applications of this method to the solution of other problems of mechanics and physics.

In this paper, we use the method of Riemann surfaces to solve one plane problem of the elasticity theory. It is related to the stressed state of a piecewise-homogeneous plane with a system of interfacial cracks and thin hard acute-angled interfacial inclusions completely detached from the media. The plane is supposed to be under the action of stresses defined at infinity and a finite number of point forces and force couples. The latter can be located both in the media and at their interface. We assume that continuations of (all or some) cracks contain glide lines whose lengths are not known beforehand. Using the modified formulae of Kolosov and Muskhelishvili, one can reduce this problem to two separate boundary-value Riemann problems for two-sheeted, generally speaking, different Riemann surfaces. The solutions of these problems, as well as the complex potentials of a compound elastic plane, can be expressed explicitly in terms of the main functionals of the surfaces and a solution of the real analog of the Jacobi problem about the inversion of Abelian integrals of the first kind on one of the surfaces. We find asymptotic representations of complex potentials in the neighborhoods of crack tips, inclusions and end points of glide lines. We use these representations in order to obtain the equations which enable us to determine the lengths of the glide lines and the analytic formulae for stress intensity factors.

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