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Approximation of Almost Periodic Functions of Two Variables

Yu. Kh. Khasanov[1*]
(Submitted by A. M. Bikchentaev)

[1] *Russian-Tajik Slavonic University, ul. M. Tursunzoda 39, Dushanbe, 734025 Republic of Tajikistan*
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Abstract—In this paper we study the deviations of periodic functions of two variables from the integral mean values and their Fourier transforms. In the class of uniform almost periodic functions of two variables we obtain estimates for the deviation from sums of Marcinkiewicz—Zygmund type.

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J. Marcinkiewicz [1] was first to consider the behavior of sums in the form

$$\sigma_n(f;x,y)=\frac{1}{n+1}\sum_{k=0}^{n}S_{k,k}(f;x,y),$$

where $f(x,y)\in L_p$ $(1\le p\le\infty)$ and the symbol $S_{k,k}(f;x,y)$ denotes partial sums of order k in each of variables of the Fourier series of the function f. In particular, he proved that if a function $f(x,y)$ is continuous in the aggregate of variables, then

$$R_n(f)_{L_\infty}=\|f(x,y)-\sigma_n(f;x,y)\|_{L_\infty}\to 0,\ \ n\to\infty.$$

Later in the paper [2] L. V. Zhizhiashvili has estimated the rate at which the value

$$R_n(f)_{L_p}=\|f(x,y)-\sigma_n(f;x,y)\|_{L_p}\ \ (1<p\le\infty)$$

converges to zero.

In [3] R. Taberski has studied the deviation of a function of two variables $f(x,y)\in L_p$ $(1\le p\le\infty)$ from sums in the form

$$W_r(f;x,y)=(1-r)\sum_{k=0}^{\infty}r^kS_{k,k}(f;x,y)$$

with $0<r<1$ and $r\to 1$.

1. Approximation of functions by some Fourier integrals. In this Item we study the deviation of functions of two variables $f(x,y)$ defined on the whole two-dimensional space from integral mean values of their Fourier transforms in the metric of the space $L_p(R^2)$ $(1\le p<\infty)$.

Let $L_p(R^2)$ $(1\le p<\infty)$ stand for the space of measurable functions $f(x,y)$ such that

$$\|f(x,y)\|_{L_p}=\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}|f(x,y)|^p dx\,dy\right]^{1/p}<\infty\ \ (1\le p<\infty)$$

and almost everywhere there exists the Fourier transform

$$F(t,z)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}f(u,v)e^{-i(tu+zv)}du\,dv,$$

*E-mail: yukhas60@mail.ru.