ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2010, Vol. 54, No. 12, pp. 1–16. © Allerton Press, Inc., 2010.
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Comparison of Numerical Integration Formulas

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Received April 24, 2009

Abstract—In this paper we study the mean square error of numerical integration, when the integrand is a random stationary process. We obtain exact asymptotic errors of classical quadrature formulas and give lower and upper bounds for the least mean square error.

DOI: 10.3103/S1066369X10120017

Key words and phrases: *quadrature formula, stationary random process.*

INTRODUCTION

An important element of the study of various numerical integration schemes is the comparison of the accuracy of algorithms on suitable classes of functions.

In this paper we study the asymptotic behavior as $n \to \infty$ of the mean square error of formulas for the numerical integration of a stationary (in a wide sense) random process (rand. proc.) $\xi(t)$, $t \in R^1$,

$$\sqrt{E\left|\int_0^1 \xi(t)dt - \sum_{k=1}^{n} c_k \xi(t_k)\right|^2}. \tag{1}$$

In other words, on the set of integrable functions we introduce some measure invariant with respect to shifts of the argument t (a "stationary measure") and then using this measure we calculate the mean square integration error (1), where the operator E is the probabilistic mean. We compare integration formulas by considering the asymptotic behavior of the mean error value (1) as $n \to \infty$ for suitable classes of random processes $\xi(t)$. In other words, we compare quadrature formulas on classes of functions by using, in particular, the mean integration error in a class rather than the worst (in the same class) functions. According to this approach, a quadrature form is said to be "better" than another one if for a wide class of stationary processes $\xi(t)$ its mean square error is less (asymptotically less).

The optimal quadrature formulas (in the sense of the mean square error) were first constructed by A. V. Sul'din [1–2] (for the Wiener measure, i.e., when $\xi(t) = W(t)$ is a standard Wiener process). It is essential to obtain the mean square error $e(n)$ of the best integration formula that corresponds to the minimum of (1) with respect to all c_k and t_k. In order to obtain the lower bounds for the mean square error, in [3] one introduced the following condition (the Sacks–Ylvisaker conditions of order r) consisting of four requirements:

(A) the covariance function $R(s,t)$ of the random process $\xi(t)$ is r times continuously differentiable in both variables and belongs to $C^{(r,r)}([0,1]^2)$;

we do not adduce items (B) and (C) here;

(D) $R^{(r,k)}(s,0) = E\xi^{(r)}(s)\xi^{(k)}(0) = 0 \quad \forall s \in [0,1],\ k = 1,2,\dots,r-1$, (condition (D) is omitted if $r = 0$).

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