ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2016, Vol. 60, No. 2, pp. 72–76. © Allerton Press, Inc., 2016.
Original Russian Text © A.B. Yakhshimuratov, M.M. Matyokubov, 2016, published in Izvestiya Vysshikh Uchebnykh Zavedenii. Matematika, 2016, No. 2, pp. 87–92.

Integration of a Loaded Korteweg-de Vries Equation in a Class of Periodic Functions

A. B. Yakhshimuratov* **and M. M. Matyokubov****

Urgench State University,
ul.Khamida Alimdzhana 14, Urgench, 220100 Republic of Uzbekistan

Received July 8, 2014

Abstract—We apply the inverse spectral problem to integrate the Korteweg-de Vries equation with a loaded term in a class of periodic functions.

DOI: 10.3103/S1066369X16020110

Keywords: *Korteweg-de Vries equation, Sturm–Liouville operator, inverse spectral problem, system of equations of Dubrovin, trace formulas.*

1. In [1–8] periodic solutions to the Korteweg-de Vries (KdV) equation were investigated. Here we consider the KdV equation with a loaded term. More precisely, consider the equation

$$q_t = q_{xxx} - 6qq_x + \gamma(t) \cdot q|_{x=0} \cdot q_x, \quad x \in R, \quad t > 0, \tag{1}$$

with the initial condition

$$q(x,t)|_{t=0} = q_0(x), \tag{2}$$

where $\gamma(t)$ is a given real-valued continuous function. We have to find a real-valued function $q(x,t)$, which is π-periodic by x, i.e.,

$$q(x+\pi,t) \equiv q(x,t), \quad x \in R, \quad t > 0, \tag{3}$$

and satisfies the following smoothness condition:

$$q(x,t) \in C_x^3(t > 0) \cap C_t^1(t > 0) \cap C(t \geq 0). \tag{4}$$

We should note that solutions to the KdV equation, with a self-consistent source, from the class of rapidly decreasing at the infinity functions were considered in [9–13], and periodic solutions to various non-linear equations with a source were investigated in [14–18].

The aim of the paper is to give a procedure of construction of a solution $q(x,t)$ to the problem (1)–(4) using the inverse spectral problem for the Sturm–Liouville operator.

2. In the Item we give some information on the inverse spectral problem for the Sturm–Liouville operator with a periodic potential [19]–[21].

Consider the following Sturm–Liouville operator:

$$Ly \equiv -y'' + q(x)y = \lambda y, \quad x \in R, \tag{5}$$

with a real-valued continuous π-periodic function $q(x)$.

Denote by $c(x,\lambda)$ and $s(x,\lambda)$ the solutions to (5), satisfying the initial conditions $c(0,\lambda) = 1$, $c'(0,\lambda) = 0$ and $s(0,\lambda) = 0$, $s'(0,\lambda) = 1$. The functions $\Delta(\lambda) = c(\pi,\lambda) + s'(\pi,\lambda)$ is called the Lyapunov function or the Hill discriminant.

The spectrum of operator (5) is pure continuous and coincides with the set

$$E = \{\lambda \in R : -2 \leq \Delta(\lambda) \leq 2\} = [\lambda_0, \lambda_1] \cup [\lambda_2, \lambda_3] \cup \cdots \cup [\lambda_{2n}, \lambda_{2n+1}] \cup \cdots.$$

*E-mail: albaron@mail.ru.
**E-mail: mmm2210410@mail.ru.