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SOME CLASSES OF SINGULAR INTEGRAL EQUATIONS SOLVABLE IN A CLOSED FORM

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At present time, rather a small number of types of singular integral equations are known possessing solutions in a closed form. This situation is related first of all to the fact that the method for solving these equations is based on reduction of them to an equivalent problem of linear conjugation, which, in its turn, must admit an efficient way of solving. However, the class of such problems is rather narrow. In this article we consider a singular integral equation equivalent to the homogeneous problem of linear conjugation with a matrix function (m. f.) of a special form; this integral equation is assumed to possess a way for a "slight modification" such that the corresponding m. f. admits an effective factorization. It is shown that for "modifying factors" of a certain form one can effectively factorize also the m. f. which corresponds to the initial equation; therefore its solution can be written in a closed form.

Let Γ be a simple smooth closed contour dividing the plane of the complex variable into the two domains D^+ and D^- ($0 \in D^+$, $\infty \in D^-$). In [1] on Γ the following singular integral equation was considered:

$$K\varphi(t) \equiv a_0(t)\varphi(t) + a_1(t)S[k_1\varphi](t) + \cdots + a_n(t)S[k_n\varphi](t) = f(t), \qquad (1)$$

whose right-hand side was taken in the form $f(t) = 2M_1(t)a_1(t) + \cdots + 2M_n(t)a_n(t)$ ($M_k(t)$, $k = \overline{1,n}$, are polynomials), $S[\omega](t)$ stands for a singular operator, while $a_0(t)$, $a_i(t)$, $k_i(t)$, $i = \overline{1,n}$, are H_μ-continuous functions of the points of the contour. If, for $i \neq j$, $i,j = \overline{1,n}$, $a_i(t) \not\equiv a_j(t)$, $k_i(t) \not\equiv k_j(t)$, $t \in \Gamma$, then equation (1) under the condition of its normal solvability is equivalent to the homogeneous problem of linear conjugation with the m. f.

$$G = G_0 - E, \quad G_0 = \|g_{ij}\| \quad (g_{ij} = 2k_i a_j/(a_0 + \Delta), \quad i,j = \overline{1,n}, \quad \Delta = k_1a_1 + \cdots + k_na_n) \qquad (2)$$

and is called singular integral equation with n kernels. It was shown that the adjoint equations with n kernels are equivalent to adjoint problems of linear conjugation and was noted that the presence of zeros of coefficients of the equation on contour does not change the picture of its solvability. The following equation $K_\alpha\varphi(t) = f_\alpha(t)$ of the form (1) was also considered, in which the coefficients a_i, k_i, $i = \overline{1,n}$, were replaced for a_i/α_i and $k_i\alpha_i$, respectively, where $\alpha_i(t)$, $i = \overline{1,n}$, are H_μ-continuous functions of contour's points. Such an equation is equivalent to the homogeneous problem of linear conjugation with the m. f.

$$G_\alpha = \operatorname{diag}\{\alpha_1, \dots, \alpha_n\} G \operatorname{diag}\{\alpha_1^{-1}, \dots, \alpha_n^{-1}\}. \qquad (3)$$

Let us distinguish the particular cases of equation (1), for which we succeed to factorize effectively both the m. f. (3), chosen in a special way, and also the m. f. of the problem of linear conjugation for equation (1).