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STABILITY BOUNDARIES OF DIFFERENCE SCHEMES

A.V. Gulin and A.V. Sheredina

1. *Introduction.* We consider two-layer and three-layer operator-difference schemes

$$By_t + Ay = 0, \tag{1}$$

$$By_{\overset{\circ}{t}} + \tau^2 R y_{\bar{t}t} + Ay = 0, \tag{2}$$

where $y = y_n = y(t_n) \in H$ are functions of a discrete argument $t_n = n\tau$ with values in a finite-dimensional linear space H, $n = 0, 1, \dots$, $\tau > 0$, and A, B, R linear operators acting in H. Here we denote

$$y_t = \frac{y_{n+1} - y_n}{\tau}, \quad y_{\bar{t}} = \frac{y_n - y_{n-1}}{\tau}, \quad y_{\overset{\circ}{t}} = \frac{y_{n+1} - y_{n-1}}{2\tau}.$$

The theory of stability of difference schemes (1), (2) was exposed in [1], [2]. In [3]–[6] criteria for stability of two-layer and three-layer difference schemes with operator weight factors were obtained, i. e., schemes of the form

$$y_t + \sigma A y_{n+1} + (E - \sigma) A y_n = 0, \tag{3}$$

$$y_{\bar{t}t} + \sigma A y_{n+1} + (E - 2\sigma) A y_n + \sigma A y_{n-1} = 0, \tag{4}$$

where σ is an operator in H, E is the unit operator. The theory of difference schemes with operator factors has its proper peculiarities and is exposed in [6] with sufficient details.

The conditions for stability obtained in [3]–[5] can be characterized by impossibility to weaken them at the expense of the choice of a norm. Let us give theorems from [3], [5], which are necessary in what follows. It is assumed that the functions A, B, and R do not depend on the number of the layer n. The stability is investigated in the norms $\|y\|_D = \sqrt{(Dy, y)}$, generated by a selfadjoint positive operator $D : H \to H$. In what follows by a *stability in the space* H_D we understand the nonincreasing with time $t = t_n$ of the norm of the solution $\|y(t_n)\|_D$ of the difference problem.

Theorem 1. *Let* $A^* = A$, $\sigma^* = \sigma$. *If scheme* (3) *is stable in a certain space* H_D, *then the operator inequality takes place*

$$A + \tau A \mu A \geq 0, \tag{5}$$

where $\mu = \sigma - 0.5E$. *Vice versa, if* (5) *is fulfilled, then scheme* (3) *is stable in* H_{A^2}.

Theorem 2. *Let* $A^* = A$, $\sigma^* = \sigma$. *If scheme* (4) *is stable in a certain space* H_D, *then the operator inequality is fulfilled*

$$A + \tau^2 A \mu A \geq 0,$$

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