ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2007, Vol. 51, No. 6, pp. 75–79. © Allerton Press, Inc., 2007.
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Quadrature Solution Methods for Nonlinear Singular Integral Equations

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Received May 12, 2006

DOI: 10.3103/S1066369X07060096

1. INTRODUCTION

This paper is dedicated to the quadrature methods, solving nonlinear singular integral equations (NSIE) in the form

$$Kx \equiv \frac{1}{\pi}\int_{-1}^{1}\frac{x(\tau)}{(\tau-t)\sqrt{1-\tau^2}}\,d\tau + \frac{\lambda}{\pi}\int_{-1}^{1}\frac{h(t,\tau,x(\tau))}{\sqrt{1-\tau^2}}\,d\tau = y(t), \quad -1<t<1, \tag{1}$$

under the additional assumption

$$\int_{-1}^{1}\frac{x(t)}{\sqrt{1-t^2}}dt = 0. \tag{2}$$

Here $y(t)$ and $h(t,\tau,u)$ are given functions defined for $-1\le t,\tau\le 1$, $-\infty<u<\infty$; λ is a numerical parameter; $x(t)$ is the desired function; the singular integral is understood in the sense of the Cauchy principal value ([1], Chap. 2, § 1). Many problems important for applications can be reduced to these NSIE on an open contour (see, e. g., [1–3] and references therein). In what follows, we consider several computational schemes of the method of mechanical quadratures. These schemes are based on the approximation of the singular operators by finite-dimensional ones generated by the quadrature formulas. The quadrature methods are most simple for the numerical realization, but their theoretical substantiation is rather difficult. To this end, we apply certain results from nonlinear functional analysis (for instance, [4]) and from the theory of singular integral equations [5, 6, 2].

2. COMPUTATIONAL SCHEMES OF THE METHOD OF MECHANICAL QUADRATURES (M. M. Q.)

Let functions $y(t)$ and $h(t,\tau,u)$ be continuous in their domains. Using the results obtained in [2] (P. 113), let us first describe the computational schemes of the m. m. q.

Scheme A. We seek for an approximate solution to problem (1)–(2) in the form

$$x_n(t) = \sum_{k=1}^{n}\alpha_k T_k(t), \quad -1<t<1, \ \ n\in N,$$

where $T_k(t) = \cos k \arccos t$ is the Chebyshev polynomial of the first kind, and condition (2) with $x_n(t)$ is fulfilled. We find the unknown coefficients α_k, $k=\overline{1,n}$, from the following system of nonlinear algebraic equations (SNAE):

$$\sum_{k=1}^{n}\alpha_k U_{k-1}(t_j) + \frac{\lambda}{n+1}\sum_{r=0}^{n}h\Big(t_j,\tau_r,\sum_{k=1}^{n}\alpha_k T_k(\tau_r)\Big) = y(t_j), \quad j=\overline{1,n},$$