

## OPTIMIZATION OF SEMI-LINEAR HYPERBOLIC SYSTEMS WITH SMOOTH BOUNDARY CONTROLS

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Reduction of inverse problems of the Mathematical Physics to problems of optimal control is a popular technique for solving them. The coefficients to be determined, elements of either right sides or initial-boundary value conditions of partial equations can be assumed as control actions. However, in a number of practical problems, the unknown parameters are functions possessing a smoothness of sufficient degree. This requirement follows from the physical sense of problems under investigation. At the same time, sufficiently powerful methods of optimal control, based on the use of the Pontryagin maximum principle, its consequences and modifications, are oriented to classes of discontinuous controls. Therefore the problem of development of methods for solving problems of optimal control in class of smooth control actions is actual with regard for constraints on the controls, which are characteristic for the inverse problems of the Mathematical Physics.

In this article we develop one of the directions of such an approach on the example of the problem of optimal control of the initial-boundary value conditions of the semi-linear hyperbolic first order system.

The peculiarity of this class of problems is the non-validity of the optimality conditions in the form of the classical (pointwise) Pontryagin maximum principle for these problems. Attention to this fact was attracted, for example, in [1], where in a special way for the simplest version of hyperbolic systems a counterexample was constructed. In [2] the differential (linearized) maximum principle was proved. In [3], a nonclassical condition of optimality in the form of the variational maximum principle was obtained. The optimal boundary control supplies the maximum value in special problems of control of initial conditions of a system of ordinary differential equations, which is constructed on the characteristics of the initial hyperbolic system.

In this article the problem of optimal control of initial-boundary value conditions of semi-linear hyperbolic equations is considered in the class of smooth control actions, restricted by pointwise (amplitude) or integral constraints. Namely this class of functions is distinctive for inverse problems of the Mathematical Physics. The problem is investigated by means of nonstandard interior variations of admissible controls. In [4], this type of variations was used to obtain the necessary conditions for the optimality in the problem of optimal control of systems of ordinary differential equations with delay.

Application of interior variations for the class of problems under investigation leads us to a new condition of optimality and allows us to create a constructive version of a method of improvement of admissible controls. The numerical realization of the method is carried out for the illustrative example of determination of the initial profile of a wave by known data of observation at the terminal moment of time.

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