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CONSTRUCTION OF ALMOST PERIODIC SOLUTIONS OF BOUNDARY VALUE PROBLEMS ON PROPAGATION OF SURFACE WAVES

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The problems on propagation of surface waves of ideal liquid were considered in [1], where they were solved by means of the integral Fourier transformation. Solutions of the problems were expressed via the inverse Fourier transformation, which represents an improper integral of oscillating functions, which leads to significant computational difficulties.

By means of a generalized discrete Fourier transformation, which was introduced and studied in author's papers [2] and [3], in this article we construct the almost periodic (in the Bohr sense) solutions of a series of problems on propagation of surface waves. These solutions are obtained in the form of the absolutely converging Fourier series whose coefficients are expressed via given functions.

By an almost periodic polynomial we call a function $p(t)$, $-\infty < t < +\infty$, which is a linear combination of functions of the form $\exp(i\lambda t)$, where $\lambda \in \mathbb{R}$. We denote by Π_C the closure with respect to norm in $L_\infty(-\infty,+\infty)$ of a set of all almost periodic polynomials. The set Π_C is a subalgebra in $L_\infty(-\infty,+\infty)$, which consists of all functions which are almost periodic in the Bohr sense. We denote by Π_W a subset in Π_C, which consists of functions $A(t) \in \Pi_C$ of the form $A(t) = \sum\limits_{n=1}^{\infty} a_n e^{i\lambda_n t}$, satisfying the condition $\sum\limits_{n=1}^{\infty} |a_n| < \infty$. The set Π_W is a Banach algebra (see [4]).

To every function $A(t)$ in Π_W we put into correspondence a function

$$a(\lambda) = M\{A(t)e^{-i\lambda t}\} = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} A(t)e^{-i\lambda t}dt. \tag{1}$$

Such a function exists and can differ from zero on at most countable set of values $\lambda : \lambda_1, \lambda_2, \dots$; $a(\lambda_n) = a_n \neq 0$ (see [5]). Thus, to every function in Π_W we put into correspondence a function $a(\lambda)$ or a sequence of pairs $a(\lambda) = \{(a_1,\lambda_1),(a_2,\lambda_2),\dots\}$, $a_n \in \mathbf{C}$, $\lambda_n \in \mathbb{R}$.

If $A(t) \in \Pi_W$, then the corresponding to this function sequence $\{a_k\}$ belongs to l_1 (we will say $a(\lambda) \in l_1$). Vice versa, for every function $a(\lambda) \in l_1$ a function $A(t)$ exists for which (1) is fulfilled and

$$A(t) = \sum_{n=1}^{\infty} a_n e^{i\lambda_n t}. \tag{2}$$

The series converges absolutely and uniformly for $-\infty < t < \infty$. Consequently, we have established a one-to-one correspondence between the functions from Π_W and the two-dimensional sequences $a(\lambda) \in l_1$. In addition, we assume that two sequences $a(\lambda), b(\lambda) \in l_1$ coincide if the same functions correspond to them, i.e., the sequence $a(\lambda)$ does not change if we add a countable set of pairs of the form $(0,\lambda)$.

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