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Stability and Unstability of Quasilinear Systems of Integrodifferential Equations

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Consider the system

$$x' = Ax + \int_0^t K(t-s)x(s)ds + f(t,x) + \int_0^t G[t,s,x(s)]ds, \qquad (1)$$

where A is a constant $n \times n$-matrix, $K \in L_1[0,\infty)$, f and G are continuous for $0 \le s \le t < \infty$, $\|x\| \le r$, and $f(t,0) = G(t,s,0) \equiv 0$.

Definitions of stability and asymptotic stability of a trivial solution to system (1) are analogous to the corresponding definitions for differential equations.

Below we establish conditions, ensuring the stability of a trivial solution to system (1); we use them later, considering one immunological model.

Let

$$\sup_t \|f(t,x)\| = o(\|x\|), \;\; x \to 0 \quad \text{and} \quad \sup_t \int_0^t \sup_{\|x\|\le\tau} \|G(t,s,x)\|ds = o(\tau), \;\; \tau \to 0. \qquad (2)$$

Theorem 1. *If the matrix $zI - A - \widehat{K}(z)$ is invertible for $\operatorname{Re} z \ge 0$ and assumptions (2) are valid, then the trivial solution to system (1) is stable. Here $\widehat{K}(z)$ stands for the Laplace transformation of the kernel K.*

If, in addition,

$$\lim_{t\to\infty} \int_0^T \sup_{\|x\|\le\tau} \|G(t,s,x)\|ds = 0 \qquad (3)$$

for any $T > 0$ and $\tau \le r$, then the trivial solution to system (1) is asymptotically stable.

The scheme of the proof. Since $\det(zI - A - \widehat{K}(z)) \ne 0$ for $\operatorname{Re} z \ge 0$, we obtain that (see, e.g., [1]) the Cauchy matrix of the linear system $x' = Ax + \int\limits_0^t K(t-s)x(s)ds$ satisfies the estimates $\|C(t)\| \le M$, $\int\limits_0^\infty \|C(t)\|dt \le M$.

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