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# The Boundary-Value Problem for Degenerate Ultraparabolic Equations of the Sobolev Type

**N. R. Pinigina**[1*]

[1] *North-Eastern Federal University, ul. Belinskogo 58, Yakutsk, 677000 Russia*

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**Abstract**—In this paper we study the first boundary-value problem for a class of degenerate equations of the Sobolev type and prove existence and uniqueness theorems for regular solutions to the considered problem.

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The Sobolev-type equations with an isolated (time) variable $t$ were studied in papers [1–4], where one has proposed methods for proving the existence of regular solutions.

In this paper we consider the boundary-value problems for degenerate equations of the Sobolev type with several isolated variables, i.e., “ultraparabolic” equations. As in [1–4], the main goal of this paper is to prove the existence of regular solutions.

Since the goal of this paper is to obtain sufficient (non-minimal) conditions guaranteeing the existence of regular solutions to the boundary-value problem for the considered equations of the Sobolev type, we assume that the coefficients of all operators mentioned below are as smooth as we need.

We restrict ourselves to studying certain model cases. In particular, we consider equations with two isolated (time) variables, because the difference between this case and the case of three or more isolated variables consists only in the cumbersome but inessential details.

Let $\Omega$ be a bounded set in the space $\mathbb{R}^n$ of variables $x_1, \dots, x_n$ with a smooth (for simplicity, infinitely differentiable) boundary $\Gamma$. Let $Q = \Omega \times (0, T_1) \times (0, T_2)$ be a cylindrical domain, $0 < T_1 < +\infty$, $0 < T_2 < +\infty$, $S = \Gamma \times (0, T_1) \times (0, T_2)$. Assume that functions $a^{ij}(x)$, $b^{ij}(x)$, $i, j = 1, \dots, n$, $a_0(x)$, $b_0(x)$, and $f(x, t, \tau)$ are defined for $x \in \overline{\Omega}$, $t \in [0, T_1]$, and $\tau \in [0, T_2]$.

We define operators $A$ and $B$ as follows: the operator $A$ is a second order elliptic-parabolic operator in the form

$$Au = \frac{\partial}{\partial x_i}(a^{ij}(x)u_{x_j}) + a_0(x)u, \quad a^{ij}(x)\xi_i\xi_j \geq 0 \ \forall x \in \overline{\Omega}, \ \forall \xi \in \mathbb{R}^n, \tag{1}$$

and the operator $B$ is an elliptic operator in the same form

$$Bu = \frac{\partial}{\partial x_i}(b^{ij}(x)u_{x_j}) + b_0(x)u, \quad b^{ij}(x)\xi_i\xi_j \geq m_0|\xi|^2, \ m_0 > 0, \ x \in \overline{\Omega}, \ \xi \in \mathbb{R}^n \tag{2}$$

(here and below the summation is performed over repeated indices from 1 to $n$). We assume that

$$a^{ij}(x) = a^{ji}(x), \ b^{ij}(x) = b^{ji}(x), \ x \in \overline{\Omega}, \ i, j = 1, \dots, n. \tag{3}$$

*The first boundary-value problem*: Find a solution to the equation

$$u_t - Au_\tau - Bu = f(x, t, \tau) \tag{4}$$

[*] E-mail: n-pinig@mail.ru.