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THE METHOD OF COMPLETE QUADRATIC APPROXIMATION IN OPTIMAL CONTROL PROBLEMS

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We propose the new method for the standard optimal control problem with nonlinear generating functions. It is based on the biquadratic approximation of the objective functional and the generalized procedure of control variation and implies the consecutive improvements and modernization of the standard schemes of the second order of accuracy [1], [2].

1. Problem definition. Method of solution

Let us formulate the basic optimal control problem

$$\Phi(u) = \varphi(x(t_1)) + \int_T F(x,u,t)dt \to \min, \tag{1}$$

$$\dot{x} = f(x,u,t), \quad x(t_0) = x^0, \tag{2}$$

$$u(t) \in U, \quad t \in T = [t_0, t_1]. \tag{3}$$

We consider the set of admissible controls V which contains the piecewise continuous vector-functions $u(t)$, $t \in T$ satisfying condition (3), where U is a convex compact set.

We suppose that in problem (1)–(3)

1) the terminal function $\varphi(x)$ is doubly continuously differentiable with respect to $x \in R^n$,
2) the integrand $F(x,u,t)$ and the vector-function $f(x,u,t)$ are continuous with respect to their arguments on $R^n \times U \times T$ together with their derivatives in the aggregate (x,u) up to the second order, inclusive.

We introduce the vector adjoined variable $\psi \in R^n$ and construct the Pontryagin function

$$H(\psi,x,u,t) = \langle \psi, f(x,u,t)\rangle - F(x,u,t).$$

Within the problem definition (1)–(3), we consider two classes of problems with special features.

The bilinear problem:

1) the function $\varphi(x)$ is linear with respect to x,
2) the function $H(\psi,x,u,t)$ is bilinear as respects the aggregate (x,u).

The biquadratic problem:

1) the function $\varphi(x)$ is quadratic with respect to x,
2) the vector-function $f(x,u,t)$ is linear with respect to x,
3) the function $H(\psi,x,u,t)$ is biquadratic as respects the aggregate (x,u).

Let us introduce the approximations of the objective functional corresponding to the accuracy of the given problems.

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