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Quasistrategies in an Abstract Guidance Control Problem

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Abstract—We consider an abstract guidance control problem with elements of uncertainty, where the control procedures are identified with set-valued quasistrategies. The goal of the control consists in the guidance to the objective set under phase constraints. We construct the solvability set by the method of program iterations which is well known in the theory of differential games. We prove that the limit of the iterative procedure has the sense of the set of positional absorption (the stable bridge) introduced by N. N. Krasovskii. This limit coincides with the solvability set in the class of quasistrategies.

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INTRODUCTION

The main denotations: DG (differential game), MPI (method of program iterations), PAO (program absorption operator), s/s (subset), s/sp (subspace), TS (topological space). Along with the adduced denotations, below we use some other abbreviations; we explain their meaning each time we use it first in this paper.

One usually considers the control problems with disturbances (the latter are understood in a wide sense) within the framework of the theory of differential games (DG) ([1–7]). In the theory of DG one uses various formalizations of the game control. Namely, the formalization of N. N. Krasovskii (it assumes that the control obeys the feedback principle), the formalization in the class of quasistrategies (it goes back to [8, 9]), the formalization in the class of ε-strategies (it was proposed by B. N. Pshenichnyi), piecewise-programmed control procedures [10, 11]. Procedures of the MPI used below are based on investigations described in [12–19]. The latter are connected with the solution of DG in the positional formalization of N. N. Krasovskii and also use the apparatus of set-valued quasistrategies [20–22]. Under the natural conditions of information consistency both these formalizations appear to be equivalent with respect to the result obtained in DG. Recall that the use of the positional formalization enabled N. N. Krasovskii and A. I. Subbotin to establish the theorem of alternative [3] which is fundamental in the theory of DG. One of the mentioned versions of the MPI implies the construction of an alternative partition of the space of positions; its existence follows from the indicated theorem. In this paper we consider an abstract version of this construction in the case, when the theorem of alternative [3] has no corresponding analog (it is not obtained in the statement adduced in [3] and in other works dedicated to the theory of DG). We propose two versions of the MPI. The first one is indirect; it is analogous to the method described in [23] and is realized on the space of sets, whose elements are game positions. The second one is direct (in sense of the construction of control procedures); it is similar to the method adduced in [24–26]. We use the duality of the mentioned versions of the MPI in the form of [23, 27]. We consider the set of all initial positions with which one can successfully solve the guidance problem with an objective set M and phase constraints defined by a set N (M and N are sets in the space of positions). We prove that this set coincides with the limit of the indirect iteration procedure. In

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