

NON-COMPACTNESS MEASURES IN INVESTIGATION OF INEQUALITIES

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Introduction

In [1] (p. 74) the Banach spaces E_0 , E , and E_1 were considered, besides, $E_0 \subset E \subset E_1$, and the embedding of E_0 into E is compact. Then, for every $\varepsilon > 0$, a constant c_ε can be found such that with all $v \in E_0$ one has

$$\|v\|_E \leq \varepsilon \|v\|_{E_0} + c_\varepsilon \|v\|_{E_1}. \quad (1)$$

In [2] (p. 154) a result by Yu.A. Dubinskiĭ was exposed, where E_0 was replaced with the set \mathfrak{S} equipped with the function $M : \mathfrak{S} \rightarrow R^+$; besides, $\mathfrak{S} \subset E \subset E_1$, $M(v) \geq 0$,

$$M(\lambda v) = |\lambda| M(v) \quad (2)$$

for all $v \in \mathfrak{S}$, the set \mathfrak{S}_1 is relatively compact in E , where

$$\mathfrak{S}_k = \{v \in \mathfrak{S} : M(v) \leq k\} \quad (3)$$

for all $k \geq 0$. In these conditions for each $\varepsilon > 0$ a constant c_ε can be found such that

$$\|u - v\|_E \leq \varepsilon (M(u) + M(v)) + c_\varepsilon \|u - v\|_{E_1} \quad (4)$$

for all u, v from \mathfrak{S} .

Remark 1 (see [2], p.154). Let $\mathfrak{S} = E_0$, M be the norm in E_0 . Then inequality (4) yields inequality (1).

We introduce the quantities

$$\tau(U) = \sup_{v \in U} M(v), \quad \tilde{\tau}(U) = \inf_{v \in U} M(v).$$

Definition 1. A set $U \subset \mathfrak{S}$ is said to be M -bounded if $\tau(U) < \infty$.

Definition 2. We shall say that a set $U \subset \mathfrak{S}$ possesses the M -property if it is M -bounded and $\tilde{\tau}(U) > 0$.

Definition 3. An operator (in general, nonlinear) $A : \mathfrak{S} \rightarrow E$ is said to be M -bounded if any M -bounded set $U \subset \mathfrak{S}$ is sent by the operator A into a bounded subset of the space E .

Without assuming (2) and also that the embedding of \mathfrak{S}_k into E is compact we shall investigate the conditions for the validity of the assertion: For each $\varepsilon > 0$ a constant c_ε can be found such that

$$\|Au - Av\|_E \leq \varepsilon (M(u) - M(v)) + c_\varepsilon \|u - v\|_{E_1} \quad (5)$$

for all u, v from an arbitrary set $U \subset \mathfrak{S}$, possessing M -property.

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