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Solvability of Systems of Volterra Integral Equations of the First Kind with Piecewise Continuous Kernels

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Abstract—We construct an asymptotic approximation for solutions of systems of Volterra integral equations of the first kind with piecewise continuous kernels. We use the asymptotics as an initial approximation in the proposed method of successive approximations to the desired solutions. We prove the existence of a continuous solution depending on free parameters and establish sufficient conditions for the existence of a unique continuous solution. We illustrate the proved existence theorems with examples.

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1. INTRODUCTION

Consider the following system of integral equations:

$$\int_0^t K(t,s)x(s)ds = f(t), \;\; 0 < t \leq T. \tag{1}$$

Assume that the matrix kernel $K(t,s)$ of dimension $m \times m$ has discontinuity points of the first kind on curves $s = \alpha_i(t)$, $i = 1, \ldots, n-1$, lying in the compact $0 \leq s \leq t \leq T$. Therefore,

$$K(t,s) = \begin{cases} K_1(t,s), & 0 \leq s \leq \alpha_1(t); \\ K_2(t,s), & \alpha_1(t) < s \leq \alpha_2(t); \\ \ldots\ldots \\ K_n(t,s), & \alpha_{n-1}(t) < s \leq t, \end{cases} \tag{2}$$

$f(t) = (f_1(t), \ldots, f_m(t))'$, and $x(t) = (x_1(t), \ldots, x_m(t))'$. Matrices $K_i(t,s)$ of dimension $m \times m$ are defined; they are continuous and have continuous derivatives with respect to t in the corresponding domains $D_i = \{(s,t) \mid \alpha_{i-1}(t) < s \leq \alpha_i(t)\}$, $i = 1, \ldots, n$, $\alpha_0 = 0$, $\alpha_n(t) = t$. Functions $f_i(t)$ and $\alpha_i(t)$ have continuous derivatives, $f_i(0) = 0$, $\alpha_i(0) = 0$, $0 < \alpha_1'(0) < \alpha_2'(0) < \cdots < \alpha_{n-1}'(0) < 1$, $0 < \alpha_1(t) < \alpha_2(t) < \cdots < \alpha_{n-1}(t) < t$ for $t \in (0,T]$. It is needed to construct continuous solutions to Eq. (1) for $t \in (0,T']$, where $0 < T' \leq T$, and $\lim\limits_{t \to +0} x(t)$ may be infinite. We assume that every matrix $K_i(t,s)$, $i = 1, \ldots, n$ has a continuous differentiable with respect to t extension to the set $0 \leq s \leq t \leq T$. The homogeneous system may have nontrivial solutions. Integral systems, particularly, those of type (1)–(2) with discontinuous kernels occur in some applications (see [1–3]). Differentiating system eqrefeq1 with respect to t, in contrast to the classical case [2, 4], we obtain a new class of Volterra integral equations with functionally perturbed argument. Therefore, the standard methods are inapplicable to system eqrefeq1, and the study of systems with kernels of type (2) has a theoretical

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