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The Ortho-Diameters of Nikol'skii and Besov Classes in the Lorentz Spaces

G. A. Akishev[1*]

[1]*Karaganda State University, ul. Universitetskaya 28, Karaganda, 100028 Republic of Kazakhstan*

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Abstract—In this paper we estimate the order of approximation of S. M. Nikol'skii and O. V. Besov classes in the norm of the anisotropic Lorentz space. We also obtain bounds for ortho-diameters of these classes.

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Let $\mathbb{R}^m$ be an m-dimensional Euclidean space of points $\overline{x} = (x_1, \dots, x_m)$ with real coordinates and let $I^m = [0, 2\pi)^m$ be the m-dimensional unit cube. Let $\overline{q} = (q_1, \dots, q_m)$, $\overline{\theta} = (\theta_1, \dots, \theta_m)$, $1 \leq q_j < +\infty$, $1 \leq \theta_j < +\infty$, $j = 1, \dots, m$, be given.

Denote by $L^*_{\overline{q},\overline{\theta}}(I^m)$ the anisotropic Lorentz space of Lebesgue measurable 2π-periodic functions $f(\overline{x})$, for which the value

$$\|f\|^*_{\overline{q},\overline{\theta}} = \left[\int_0^{2\pi} t_m^{\frac{\theta_m}{q_m}-1}\left[\cdots\left[\int_0^{2\pi}\left(f^{*_1,\dots,*_m}(t_1,\dots,t_m)\right)^{\theta_1} t_1^{\frac{\theta_1}{q_1}-1}dt_1\right]^{\frac{\theta_2}{\theta_1}}\cdots\right]^{\frac{\theta_m}{\theta_{m-1}}}dt_m\right]^{\frac{1}{\theta_m}}$$

is finite, where $f^{*_1,\dots,*_m}(t_1,\dots,t_m)$ is a nonincreasing permutation of the function $|f(\overline{x})|$ with respect to each variable x_j with fixed other variables (first with respect to x_1, then with respect to x_2, etc.) ([1, 2]).

For $1 \leq p < +\infty$ denote by $L_p(I^m)$ the Lebesgue space with the norm

$$\|f\|_p = \left[\int_0^{2\pi}\cdots\int_0^{2\pi}|f(\overline{x})|^p dx_1\dots dx_m\right]^{\frac{1}{p}} < +\infty;$$

let $L_\infty(I^m)$ be the space of essentially bounded Lebesgue measurable functions with the norm ([3], P. 12)

$$\|f\|_\infty = \sup_{\overline{x}\in I^m} \operatorname{vrai}|f(\overline{x})|.$$

Let us expand functions $f \in L_1(I^m)$ such that

$$\int_0^{2\pi} f(\overline{x})dx_j = 0 \ \ \forall j = 1,\dots,m,$$

in the Fourier series

$$f(\overline{x}) \sim \sum_{\overline{n}\in\overset{\circ}{\mathbb{Z}}{}^m} a_{\overline{n}}(f)e^{i\langle\overline{n},\overline{x}\rangle},$$

where $a_{\overline{n}}(f)$ are the Fourier coefficients with respect to the multiple trigonometric system $\{e^{i\langle\overline{n},\overline{x}\rangle}\}$ and $\overset{\circ}{\mathbb{Z}}{}^m = \{\overline{k} = (k_1,\dots,k_m) \in \mathbb{Z}^m : k_j \neq 0, j = 1,\dots,m\}$.

*E-mail: akishev@ksu.kz.