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Generalization of the Notion of Grand Lebesgue Space

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Abstract—We introduce families of weighted grand Lebesgue spaces which generalize weighted grand Lebesgue spaces (known also as Iwaniec–Sbordone spaces). The generalization admits a possibility of expanding usual (weighted) Lebesgue spaces to grand spaces by various ways by means of additional functional parameter. For such generalized grand spaces we prove a theorem on the boundedness of linear operators under the information of their boundedness in ordinary weighted Lebesgue spaces. By means of this theorem we prove boundedness of the Hardy–Littlewood maximal operator and the Calderon–Zygmund singular operators in the weighted grand spaces.

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1. INTRODUCTION

T. Iwaniec and C. Sbordone [1] introduced a new type of functional spaces which are called *grand Lebesgue spaces* (or *Iwaniec–Sbordone spaces*). Operators of harmonic analysis are investigated in the spaces intensively last time; they continue to attract attention in connection with various applications [2–13].

The initial definition of the spaces assumes boundedness of measure of definition set for the functions considered. In the papers [14, 15] an approach was suggested which let to introduce Iwaniec–Sbordone spaces for sets of infinite measure. In the paper we consider a more common approach to definition of grand Lebesgue spaces by an arbitrary open set $\Omega \subseteq \mathbb{R}^n$. We consider a common weighted case which let us to know when boundedness of linear operators in usual weighted Lebesgue space implies their boundedness in constructed by appropriate way weighted grand Lebesgue space.

The grand Lebesgue spaces $L^{p)}(\Omega, w_a)$, defined here, are generated by the weighted Lebesgue spaces $L^p(\Omega, w)$ with change of power p and weight w. Therefore, they are called *generalized weighted grand Lebesgue spaces*. For sets of finite measure the introduced spaces coincide with known grand Lebesgue spaces if the functional parameter $a \equiv 1$.

The main result of the paper is a general theorem on transfer the property of boundedness of linear operators from usual weighted Lebesgue spaces to considered here weighted grand Lebesgue spaces and its application to investigation the maximal operator and the singular Calderon–Zygmund operators.

In [5] for sets of finite measure and for classical extension ($a \equiv 1$) it is proved that boundedness of the maximal operator in weighted grand Lebesgue spaces is equivalent to the Muckenhoupt condition. A similar statement for the one-dimensional singular Hilbert operator was proved in [9, 16] (see also [11]).

As it is known ([5]), weighted boundedness of an operator in a grand space is not equivalent to its boundedness in appropriate weighted space. Similar weighted boundedness for the one-dimensional singular operator was investigated in [12] and [17].

The paper has the following structure. In Section 2 we give necessary auxiliary information. In Section 3 we introduce the grand spaces $L^{p)}(\Omega, w_a)$ and making use interpolation with change of measure we prove a theorem on boundedness of linear operators in the spaces (Theorem 2). In Section 4

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