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Description of Orthogonal Vector Fields Over W^*-Algebra of Type I_2

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Abstract—In the paper we give a characterization of a w^*-continuous orthogonal vector field F over a W^*-algebra of type I_2 in terms of reductions of F on the center of the algebra. As an application we obtain a new proof of the assertion that an arbitrary w^*-continuous orthogonal vector field over a W^*-algebra of type I_2 is stationary.

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In memory of D. Kh. Mushtari

INTRODUCTION

The works by P. Masani [1, 2], who studied integration with respect to orthogonal measures defined on rings of sets, were continued in many publications on non-commutative measure theory. Some problem set-ups and approaches to solutions for such kind of problems can be found in [3, 4], [5], § 31. One of the natural problems of this sort is the problem of characterization of Hilbert-valued linear mappings of a W^*-algebra preserving the orthogonality property (so-called orthogonal vector fields). A well-known result for commutative W^*-algebra ([5], theorem 31.19) is, in a sense, a benchmark result in this area.

In this paper we give a characterization of a w^*-continuous orthogonal vector field over a W^*-algebra of type I_2 in terms of reductions of the field on the center of the algebra. As an application we obtain a new proof of the assertion that an arbitrary w^*-continuous orthogonal vector field over an algebra of type I_2 is stationary. (For arbitrary von Neumann algebras stationarity of such fields was established earlier in [4], theorem 1.8.)

1. PRELIMINARY INFORMATION

Let $\mathcal{A}$ be a W^*-algebra. By $\mathcal{A}^{\rm pr}$, $\mathcal{A}^{\rm sa}$, and $\mathcal{A}^{\rm un}$ we denote, respectively, the sets of orthoprojectors, self-adjoint, and unitary elements of $\mathcal{A}$, by $\mathrm{rp}(x)$ a *rank projector* of an element x (the smallest projector $p \in \mathcal{A}^{\rm pr}$ such that $px = x$). In this paper we will consider W^*-algebras of type I_2. It is a well-known fact that any such W^*-algebra $\mathcal{N}$ can be represented as $\mathcal{N} = \mathcal{M} \otimes M_2$, where $\mathcal{M}$ is a commutative W^*-algebra, and M_2 is an algebra of complex matrices of order 2, so the elements of the algebra $\mathcal{N}$ are the matrices of the form $(x_{ij}) = \left(\begin{smallmatrix} x_{11} & x_{12} \\ x_{21} & x_{22}\end{smallmatrix}\right)$, where $x_{ij} \in \mathcal{M}$. By the symbols $\mathbf{I}$ and $\mathbf{1}$ we will denote the units of the algebras $\mathcal{N}$ and $\mathcal{M}$, respectively. By $\{\varepsilon_{ij}\}$ we will denote a matrix unit of the algebra $\mathcal{N}$ of the form

$$\varepsilon_{11} = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & 0 \end{pmatrix}, \quad \varepsilon_{12} = \begin{pmatrix} 0 & \mathbf{1} \\ 0 & 0 \end{pmatrix}, \quad \varepsilon_{21} = \begin{pmatrix} 0 & 0 \\ \mathbf{1} & 0 \end{pmatrix}, \quad \varepsilon_{22} = \begin{pmatrix} 0 & 0 \\ 0 & \mathbf{1} \end{pmatrix}.$$

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