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Implicit Function Method in Solving a Two-Dimensional Nonlinear Spectral Problem

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1. Nonlinear spectral problems arise in various areas of analysis and mathematical physics. Most developed are the theory and methods of solving such problems with one-dimensional spectral parameter (see [1–9]).

In this paper, we consider a nonlinear spectral problem with two-dimensional spectral parameter. The use of implicit function methods allows us to study qualitative characteristics of the existing spectrum of holomorphic operator-functions and to construct comparatively simple algorithms for numerical finding of spectral lines or connected components of spectral domains.

2. Let E be a complex Banach space, and let $\mathbf{\Lambda} = \Lambda_1 \times \Lambda_2$ be an open connected set in the complex space $\mathbb{C}^2$. Elements of $\mathbb{C}^2$ will be written in the form $\boldsymbol{\lambda} = (\lambda_1, \lambda_2)$, where $\lambda_i \in \Lambda_i \subset \mathbb{C}\,(i = 1, 2)$. Let an operator-function

$$A(\lambda_1, \lambda_2) = T(\lambda_1, \lambda_2) - I$$

be given, where $T(\lambda_1, \lambda_2)$ is a linear continuous operator acting in the space E and depending on a two-dimensional parameter (λ_1, λ_2), I is the identity operator in E. Thus, to each value $\boldsymbol{\lambda} \in \mathbf{\Lambda}$, an operator $A(\lambda_1, \lambda_2) \in \mathcal{L}(E, E)$ is assigned.

In this paper, for the nonlinear spectral problem

$$A(\lambda_1, \lambda_2)x = 0, \tag{1}$$

we find eigenvalues $\boldsymbol{\lambda}^{(0)} = (\lambda_1^{(0)}, \lambda_2^{(0)}) \in \mathbf{\Lambda}$ and the corresponding eigenvectors $x^{(0)} \in E\,(x^{(0)} \neq 0)$ such that $A(\lambda_1^{(0)}, \lambda_2^{(0)})x^{(0)} = 0$.

In the case of a finite-dimensional space E, the function

$$F(\lambda_1, \lambda_2) = \det(T(\lambda_1, \lambda_2) - I)$$

is the determinant whose order coincides with the dimension of the space. In the case when $T(\lambda_1, \lambda_2)$ is a completely continuous operator acting in a functional Hilbert space, we define the function $F(\lambda_1, \lambda_2)$ as follows:

$$F(\lambda_1, \lambda_2) = \det \begin{pmatrix} t_{11}(\lambda_1, \lambda_2) - 1 & t_{12}(\lambda_1, \lambda_2) & \dots & t_{1n}(\lambda_1, \lambda_2) & \dots \\ t_{21}(\lambda_1, \lambda_2) & t_{22}(\lambda_1, \lambda_2) - 1 & \dots & t_{2n}(\lambda_1, \lambda_2) & \dots \\ \dots & \dots & \dots & \dots & \dots \\ t_{n1}(\lambda_1, \lambda_2) & t_{n2}(\lambda_1, \lambda_2) & \dots & t_{nn}(\lambda_1, \lambda_2) - 1 & \dots \\ \dots & \dots & \dots & \dots & \dots \end{pmatrix},$$

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