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Boundedness of Cesàro Mean Values for Functions from a Hardy Space in a Polydisc

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Abstract—We prove that Cesàro mean values of analytic functions in the unit polydisc in a Hardy space are bounded with respect to the parameter m, provided that the rest parameters satisfy certain conditions.

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The Hardy class $H^p(D^n)$, $p > 0$, is understood as the set of all functions f which are analytic in the unit polydisc $D^n = \{z = (z_1, \dots, z_n) \in \mathbb{C}^n : |z_j| < 1, j = \overline{1, n}\}$, and such that

$$\|f\|_p := \sup_{\substack{0 \le \rho_j < 1, \\ j=\overline{1,n}}} \|f(\rho e^{i\theta})\|_p = \sup_{\substack{0 \le \rho_j < 1, \\ j=\overline{1,n}}} \left\{ \frac{1}{(2\pi)^n} \int_{Q^n} |f(\rho e^{i\theta})|^p d\theta \right\}^{1/p} < \infty$$

with $0 < p < \infty$ and

$$\|f\|_\infty := \sup_{\substack{0 \le \rho_j < 1, \\ j=\overline{1,n}}} \|f(\rho e^{i\theta})\|_\infty = \sup_{\substack{0 \le \rho_j < 1, \\ j=\overline{1,n}}} \left\{ \max_{\theta \in Q^n} |f(\rho e^{i\theta})| \right\} < \infty$$

with $p = \infty$, where $\rho e^{i\theta} = (\rho_1 e^{i\theta_1}, \dots, \rho_n e^{i\theta_n})$, $Q^n = [-\pi, \pi]^n$, and $d\theta = d\theta_1 \dots d\theta_n$.

For a function $f \in H^p(D^n)$, S. Bochner [1] has proved that almost everywhere on the unit torus $T^n = \{z \in \mathbb{C}^n : |z_j| = 1, j = \overline{1, n}\}$ there exists a radial boundary function $f(e^{i\theta}) \in L^p_{Q^n}$, where $e^{i\theta} = (e^{i\theta_1}, \dots, e^{i\theta_n})$, while $\|f(\rho e^{i\theta})\|_p \le \|f(e^{i\theta})\|_p = \|f\|_p$ (see also [2], Chap. III, 3.4, 3.4.2).

Let $f(\rho e^{i\theta}) = \sum\limits_k \widehat{f}_k (\rho e^{i\theta})^k$ be the Taylor series for the function $f \in H^p(D^n)$. Here $k = (k_1, \dots, k_n)$, $k_j \in \mathbb{N} \bigcup \{0\}$, $j = \overline{1, n}$, $(\rho e^{i\theta})^k = (\rho_1 e^{i\theta_1})^{k_1} \cdots (\rho_n e^{i\theta_n})^{k_n}$, and $\widehat{f}_k = \widehat{f}_{k_1, \dots, k_n}$ are coefficients of the Taylor series.

Cesàro mean values (C, α), $\alpha > -1$, for a function $f \in H^p(D^n)$ are sums

$$\sigma_m^\alpha(f, e^{i\theta}) = (A_m^\alpha)^{-1} \sum_{\substack{|k| \le m \\ m \in \mathbb{N} \bigcup \{0\}}} A^\alpha_{m-|k|} \widehat{f}_k e^{ik\theta}, \tag{1}$$

where A_m^α is the mth coefficient in the binomial series, i.e., $(1-x)^{-(1+\alpha)} = \sum\limits_{m=0}^{\infty} A_m^\alpha x^m$, while $A_m^\alpha = \binom{\alpha+m}{m} = \frac{\Gamma(\alpha+m+1)}{\Gamma(m+1)\Gamma(\alpha+1)}$, and $|k| = k_1 + \dots + k_n$.

For $n = 1$ mean values (1) are studied by E. A. Storozhenko in [3]. Note that one cannot unambiguously generalize the notion of Cesàro mean values defined for the one-dimensional case onto an n-dimensional one ($n > 1$). For example, in [4] one considers Cesàro mean values different from (1). In [5] one studied Cesàro mean values in a more general form $\sigma_m^{q,\alpha}$, which coincide with (1) for $q = 1$.

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