

ON DUALITY OF DIFFERENT VERSIONS OF THE METHOD OF PROGRAMMED ITERATIONS

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For an axiomatically defined hereditary system we consider two constructions, going back to the known in the theory of differential games method of programmed iterations: a direct (in the sense of construction of controlling procedures) version and an indirect version which supplies an object important for the corresponding problem of control (price function, stable bridge), but is an intermediary in the construction of controlling procedure. We establish the duality of these methods which embraces the operators forming iteration sequences, sequences by themselves, their limits, and conditions for absence of degeneracy of both iteration procedures.

1. Introduction

Let us note some abbreviations used in what follows: DG (differential game), MPI (method of programmed iterations), MM (multivalued mapping), m/s (multiselector), DS (directed set), OPA (operator of program absorption), s/s (subset), s/sp (subspace), TS (topological space).

In the theory of DG they use methods related to construction of stable bridges in the sense of N.N. Krasovskii and the price function of the corresponding DG (see [1]–[5]). Known difficulties in solving DG on the basis of dynamical programming were noted in [6]. Development of this method for solving DG required an application of the non-smooth analysis in connection with the non-smoothness of the price function (see [4]). In so-called regular DG auxiliary program constructions, which use game analogs of the L.S. Pontryagin maximum principle, were applied (see [1], [2]). For the general case of a nonlinear DG, N.N. Krasovskii and A.I. Subbotin established the fundamental theorem on alternative (see [2]) which has numerous important consequences (see [2]–[4]). The method of program iterations (MPI) (see [7]–[11]) arose intrinsically for solving DG in general case. A survey of basic constructions of MPI was given in [5] and in [12]. Analogs of MPI were used in works on the theory of DG (we note only some works which are most close to our considerations: [13]–[17]). In addition, analogs of MPI begun to be applied for investigation of dynamical problems of another nature: Construction of generalized solutions of the Hamilton–Jacobi equation (see [18], [19]), construction of nonanticipating or hereditary m/s of MM in [20]–[23]. In the constructions of MPI methodology related to the theorems on fixed point consecutively turns to be more essential. We can note, in particular, works by L.V. Kantorovich in connection with constructions similar to those in [24] (pp. 237–239). The question arises about comparison of different versions of MPI: The version (see [8]) realized on the space of subsets of the space of positions, and the version (see [20]–[23]) which in what follows will be called the direct version.

In Section 2 we give a summary of notation of a general character. The third Section contains more special concepts related to the representation of an abstract dynamical system and the discussion of some possible concretizations. As essential role belongs to a series of conditions used in

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