COMPLEX KERR GEOMETRY AS ALTERNATIVE TO SUPERSTRING THEORY

Alexander Burinskii

NSI, Russian Academy of Sciences, Moscow, Russia

15 RUSSIAN GRAVITATIONAL CONFERENCE,

RUSGRAV-2014, June 30 - July 5, 2014, Kazan

based on:

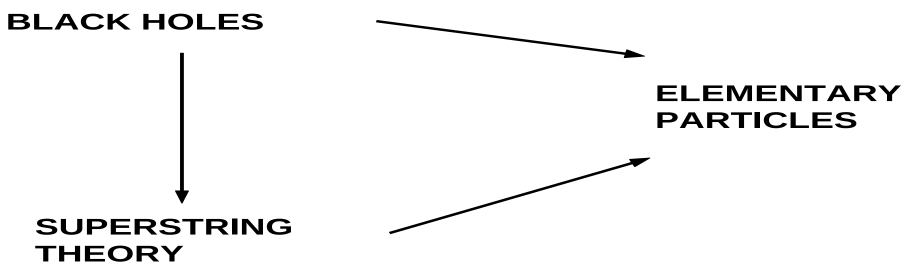
A.B., String-like structures in the 4-dim Kerr geometry: Stringy System, Kerr Theorem, and Calabi-Yau Twofold, *Adv. High Energy Phys.* 2013:509749, (2013);

A.B., String-like Structures in Complex Kerr-Schild Geometry: the $N = 2$ strings, twistors..., *Theor. Math. Phys.*, **177**(2), 1492, (2013)

BLACK HOLES - STRINGS - PARTICLES

It is broadly discussed that black holes are related with elementary particles and string theory ['t Hooft (1990), A.Salam and J. Strathdee (1976), Witten (1992), C.F.E. Holzhey and F. Wilczek (1992), A. Sen (1995), at al.].

PARTICLES and STRINGS



"... **realistic model of elementary particles still appears to be a distant dream.**" (J. Schwarz, arXiv:1201.0981)

KERR GEOMETRY corresponds to background of an electron!

Measurable parameters of an electron (mass, spin, charge, magnetic moment) indicate that its gravitational and electromagnetic field correspond to Kerr-Newman solution.(Carter 1968, Israel 1970, AB 1974, López 1984...)

BLACK HOLES and PARTICLES

Spin of particles is extreme high, black hole horizons disappear:
spin/mass ratio, $J/m > 10^{20}$ (units $G = \hbar = c = 1$) $\Rightarrow a = J/m >> m.$

SPIN of particles is extreme high: The horizons condition $m > a$. Indeed we have a / m = 10^{44}.

OVER-ROTATING KERR GEOMETRY – WITHOUT HORIZONS

FUNDAMENTAL STRINGS are soliton like solutions to low-energy string theory. Some solutions to Einstein's eqs. are exact solutions to effective string theory.

NAKED SINGULAR RING AS A CLOSED STRING

(AB, Ivanenko 1975, AB 1974). Fundamental string solutions to low-energy string theory (Witten 1985, Horowitz & Steif 1990, Sen 1992, AB 1995.) Strings as Solitons & Black Holes as Strings, (Dabholkar at al 1995).

COMPLEX STRING STRUCTURES APPEAR IN COMPLEX KERR GEOMETRY! **(AB arXiv:gr-qc/9303003, arXiv:hep-th/9503094)**

Calabi-Yau twofold inside the Kerr geometry (AB, arXiv:1307.5021)

Recently, the real and complex strings were independently (?) reobtained by Adamo and Newman (Adamo&Newman, PRD 2011).

"...It would have been a cruel god to have layed down such a pretty scheme and not have it mean something deep." **(Adamo&Newman, arXiv:1101.1052, PRD 2011).**

Kerr's gravity appears as a BRIDGE between particles and strings:

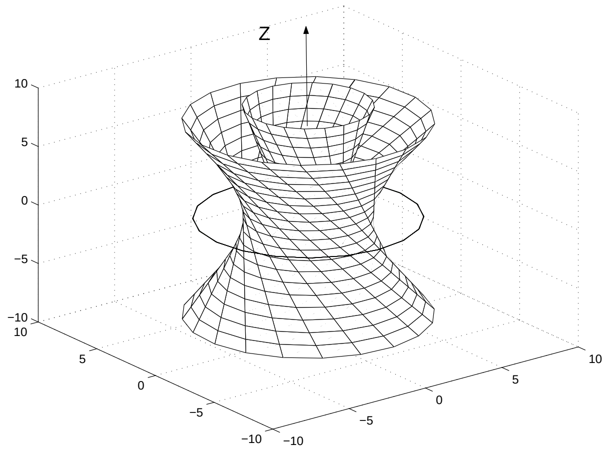
Spinning Particles ↔ Kerr's Gravity ↔ String theory

Twosheeted topology. Stringy defect of space-time.

Metric

$$g_{\mu\nu} = \eta_{\mu\nu} + 2Hk_\mu k_\nu, \quad H = \frac{mr - e^2/2}{r^2 + a^2\cos^2\theta}. \tag{1}$$

The Kerr singular ring



REGULARIZATION of SPACE-TIME
SOLITON SOURCE OF the KERR-NEWMAN SOLUTION SHOULD REPLACE KERR SINGULARITY FOR AGREEMENT with FLAT SPACE of QUANTUM THEORY

VACUUM BUBBLE (similar to MIT-bag and SLAC-bag). SHAPE is determined by eq. $H = 0$. **!!!**
Emergence of the Dirac equation (see section talk).

KERR's STRINGY SYSTEM

Second string appears in *complex* structure of Kerr geometry, (AB 1993).

TWISTOR STRUCTURE OF THE KERR GEOMETRY. *Inherent Calabi-Yau space appears as a quartic in the projective twistor space* CP^3 .

The closed Kerr string and open complex string form together 4D string-membrane system , which is parallel with string/M-theory unification (AB, arXiv:1211.6021).

PROPOSITION: Emergence of this similarity is the N = 2 superstring, structure of which is remarkable similar to structure of COMPLEX SOURCE OF KERR GEOMETRY!

N = 2 SUPERSTRING, (M.Geen, J.Schwarz and E.Witten, *Superstring Theory* V.1.)

"...N = 2 extension of the superstring construction gives a highly symmetric two-dimensional theory an interesting generalization of the super-Virasoro algebra. It seemingly cannot be given the usual interpretation of a string theory... Perhaps it enters physics in some other and yet unknown way... ... crucial subtleties in this theory have not yet been unraveled."

$Z^\mu = X^\mu + iY^\mu, \quad \mu = 0, 1 \Rightarrow$ ***four real dims!*** (Ooguri-Vafa 1990, Gibbons at al, D'Adda-Lizzi)

$$S = -\frac{1}{2\pi} \int d^2\sigma \{ \partial_\alpha Z \partial^\alpha \bar{Z} - i\bar{\psi}\gamma^\alpha \partial_\alpha \psi \}$$

The global N =2 supergauge transformations

$$\delta Z = \bar{\epsilon}\psi, \quad \delta\psi = -i\gamma^\alpha \epsilon \partial_\alpha Z \tag{2}$$

"...there are no transverse oscillations at all... the massless scalar ground state is the only propagating degree of freedom...(at least for this sector). However, subtleties in the quantization...have been pointed out recently, and this statement may require revision."

Complex String in 4D complex Kerr geometry (A.B., *String-like Structures in Complex Kerr Geometry,* [arXiv:gr-qc/9303003])

Recently, this structure were noted by Adamo&Newman (PRD 2011): **"...It would have been a cruel god to have layed down such a pretty scheme and not have it mean something deep."**

Proposition: ***complex source of Kerr geometry is analog of N = 2 string!***

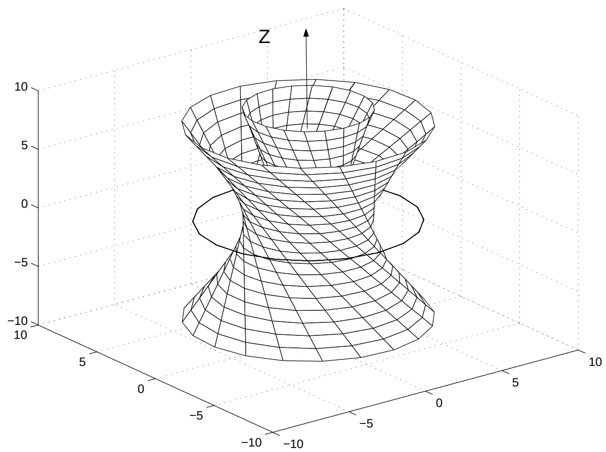
REAL structure of the Kerr-Newman solution: Metric

$$g_{\mu\nu} = \eta_{\mu\nu} + 2Hk_\mu k_\nu, \quad H = \frac{mr - e^2/2}{r^2 + a^2\cos^2\theta}, \tag{3}$$

and electromagnetic (EM) vector potential is

$$A^\mu_{KN} = Re\frac{e}{r + ia\cos\theta}k^\mu. \tag{4}$$

Gravitational and EM fields are concentrated near the Kerr singular ring.



The Kerr ring forms a branch line of space. The KN geometry is TWOSHEETED! Vector field $k_\mu(x)$ is tangent to Principal Null Congruence (PNC),

$$k_\mu dx^\mu = P^{-1}(du + \bar{Y}d\zeta + Yd\bar{\zeta} - Y\bar{Y}dv), \quad Y(x) = e^{i\phi}\tan\frac{\theta}{2}, \tag{5}$$

where $Y(x)$ is projective angular coordinate, and

$$\zeta = (x + iy)/\sqrt{2},\ \bar{\zeta} = (x - iy)/\sqrt{2},\ u = (z - t)/\sqrt{2},\ v = (z + t)/\sqrt{2}$$

are the null Cartesian coordinates.

Kerr congruence is controlled by the

KERR THEOREM:

The geodesic and shear-free Principal null congruences (type D metrics) are determined by holomorphic function $Y(x)$ which is analytic solution of the equation

$$F(T^a) = 0 \ , \tag{6}$$

where F is an arbitrary analytic function of the ***projective twistor coordinates***

$$T^a = \{Y, \quad \zeta - Yv, \quad u + Y\bar{\zeta}\}. \tag{7}$$

The Kerr theorem is a practical tool for obtaining exact solutions:

$$F(T^a) = 0 \Rightarrow F(Y, x^\mu) = 0 \Rightarrow Y(x^\mu) \Rightarrow k^\mu(x)$$

For the Kerr-Newman solution function F is quadratic in Y, which yields TWO roots $Y^{\pm}(x) \Rightarrow$ two different congruences at the same background! **Twosheeted geometry!** Functions $F(T^a)$ of higher degrees in Y correspond to **multi-sheeted geometry** and multi-particle solutions, [AB (2006)].

Complex Structure of the Kerr geometry.

Complex Shift. Appel solution 1887!

A point-like charge e, placed on the complex z-axis $(x_0, y_0, z_0) = (0, 0, -ia)$, gives a real potential

$$\phi_a = Re \frac{e}{r + ia\cos\theta}, \quad (8)$$

r and θ are oblate spheroidal coordinates.

Complex light cones with the vertexes on the ***complex world-line*** $x_0^\mu \in CM^4$:

$(x_\mu - x_{0\mu})(x^\mu - x_0^\mu) = 0$

splits into families of the "left" and "right" complex null planes:

$x_L^\mu = x_0^\mu(\tau) + \alpha e^{1\mu} + \beta e^{3\mu}$ spanned by null vectors e^1 and e^3,

and $x_R^\mu = x_0^\mu(\tau) + \alpha e^{2\mu} + \beta e^{3\mu}$, spanned by $e^2 = \bar{e}^1$ and e^3.

The Kerr congruence $\mathcal{K}$ arises as a real slice of the family of the "left" null planes ($Y = const.$) of the complex light cones with vertices at a complex world-line $x_0(\tau)$.

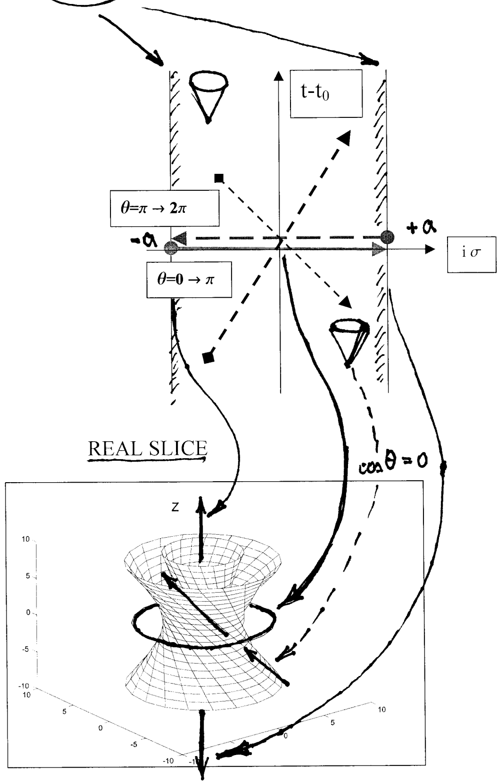
The complex light cones, presented in spinor form $\mathcal{K}_L = \{x : x = x_L^i(\tau_L) + \psi_L^A \sigma^i_{A\dot{A}} \tilde{\psi}_R^{\dot{A}}\}$. are split into two families of null planes: "Left" (ψ_L =const; $\tilde{\psi}_R$ -var.) and "Right"($\tilde{\psi}_R$ =const; ψ_L -var.).

Kerr's source can be considered as a mysterious "particle" propagating along a *complex world-line* $x_0^\mu(\tau)$ in CM^4, parametrized by a complex time τ.

Complex World line as a Complex String $X_\mu(\tau)$

Plane of the Complex time $\tau = t + i\sigma = t + i\, a \cos\theta$

End points of the open string $\sigma = -a, +a \Leftrightarrow \theta = -\pi, +\pi$



Complex open string. The complex world line (CWL) $x_0^\mu(\tau)$, parametrized by complex time τ, represents a two-dimensional surface which takes an intermediate position between particles and strings(Ooguri & Vafa 1991, AB 1993). The corresponding "hyperbolic string" equation

$$\partial_\tau \partial_{\bar\tau} x_0(t,\sigma) = 0 \qquad (9)$$

corresponds to bosonic part of the complex N=2 string. The general solution $x_0(t,\sigma) = x_L(\tau) + x_R(\bar\tau)$ is a sum of the analytic and anti-analytic modes $x_L(\tau)$, $x_R(\bar\tau)$. For each real point x^μ, the parameters $\tau = \tau_L$ and $\bar\tau = \tau_R$ should be determined by a *complex retarded-time construction*, a complex analog of the real one.

Existence of the real slice requires the **complex string has to be open**Complex world-line forms the world-sheet of an *open* with the end points at $\sigma = \pm a$.

World sheet orientifold. It is impossible to introduce the same boundary conditions for the real and imaginary part of the complex string. The problem is resolved by orientifolding the world sheet (AB, gr-qc/9303003), which forms from the open string a folded closed string. The world-sheet parity transformation $\sigma \to -\sigma$ reverses orientation of the world sheet, and covers it second time in mirror direction. Simultaneously, the Left and Right modes are exchanged. Two oriented copies of the interval $\Sigma = [-a,a]$, $\Sigma^+ = [-a,a]$, and $\Sigma^- = [-a,a]$ are joined, forming world-sheet of a closed folded string, $S^1 = \Sigma^+ \bigcup \Sigma^-$, parametrized by $\sigma = a\cos\theta$, which covers the world-sheet twice.
Orientifold puts the restriction $x_L(\tau) = x_R(\bar\tau)$.

The real Kerr geometry is formed by the Left and Right complex world-lines (CWL). The complex light cone is split into the Left and Right complex null planes. In accord to the retarded-(advanced)-time equations $\tau^{\mp} = t \mp \tilde{r}$, intersections of the Left null planes with Left CWL, together with the conjugate Right structure, determine four retarded-advanced complex time parameters

$$\tau_L^{\mp} = t \mp (r_L + ia\cos\theta_L) \quad (10)$$
$$\tau_R^{\mp} = t \mp (r_R + ia\cos\theta_R). \quad (11)$$

Along with the considered complex world-line (say 'Left'), there is a complex conjugate world-line, $X_L(\tau_L)$ and $X_R(\tau_R)$.

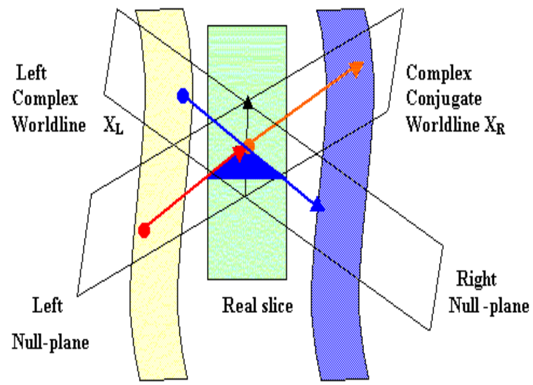


Figure 1: Complex light cone at a real point x. The adjoined to congruence Left and Right complex null planes. Four roots: X_L^{adv}, X_L^{ret} and X_R^{adv}, X_R^{ret} which are related by crossing symmetry.

orientifold projection $\Omega = Antipodalmap + Compl.Conj. + Revers\ of\ time.$

By excitations, the Left and Right structures should be considered as independent and generated by different KN sources $\Rightarrow$, which corresponds to two-particle KN system with *quadratic* generating functions of the Kerr theorem $F_1(T)$ and $F_2(T)$, determined on the projective twistor space CP^3.

The joint twistor system is described by the equation $F_{12}(T) = F_1(T) \cdot F_2(T) = 0$, which is *QUARTIC* in the projective twistor space, and therefore, the complex string forms a *Calabi-Yau twofold (K3 surface) in the projective twistor space* [arXiv:1203.4210].

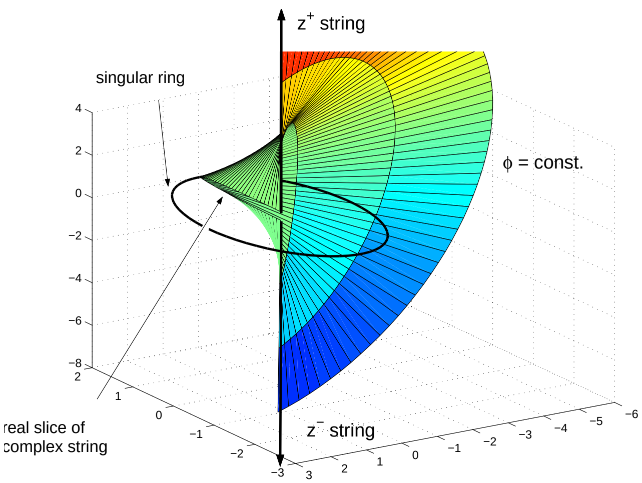


Figure 2: One sheet of the K3 for $r > 0$ and $\phi = const.$ Kerr congruence is tangent to singular ring at $\theta = \pi/2$.

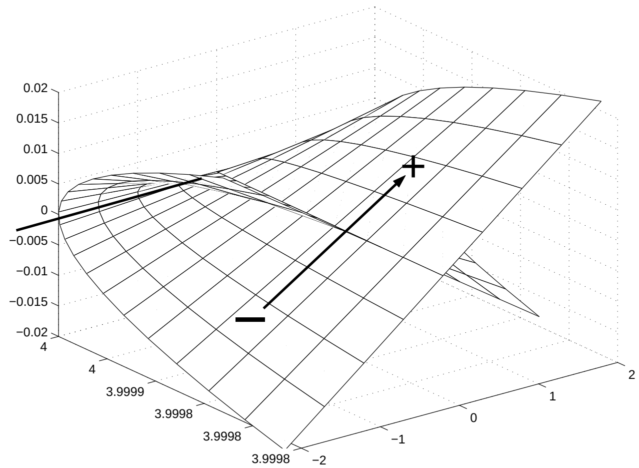


Figure 3: Section of the K3 near the Kerr ring corresponding to $\phi = const.$. Two sheets ($r > 0$ and $r < 0$) form a Möbius strip covering the space-time twice.

Embedding of the N = 2 superstring. The complex N=2 string has the real critical dimension D=4. It is one of the three consistent critical strings (D=4, D=10 and D=26).
It could be used as a basis of some four-dimensional string theory. However, there was a **principal obstacle for its application, which emerged from the attempts of its embedding in the real minkowskian space-time.** The **real embedding** may only be done for (2,2) or (4,0) signature.
The problem of signature disappears by **embedding in the complex 4D Kerr geometry**, where diverse sections have different signatures, and in particular, there exists the real minkowskian slice.
The N=2 superstring was first considered in the series of three papers by Ademollo at al. in 1976. The complex SU(2) version of this string was discussed in the *third paper* of this series in the **real world-sheet coordinates**. Unfortunately, we do not know **which subtleties** of the N=2 string were implied in the GSW book. Probably, it is necessity of the *orientifold construction*, which untangles the problem of boundary conditions. However, orientifold was invented much later in the paper by L. Dixon,J.A.Harvey, C.Vafa, E.Witten (Nucl.Phys. 1987).
Fermionic part of the N = 2 superstring (the Dirac spinor) plays important role fixing the Left null planes of twistorial structure of the complexified 4d Kerr geometry.
Therefore, N = 2 superstring may consistently be embedded in the complex 4D Kerr geometry, playing the role of its complex source.

Conclusion.

Striking parallelism with superstring/M-teory theory. Product of the KN closed heterotic string on the KN complex string creates the M2-brane – which corresponds to the relativistically rotating string-membrane source of KN spinning particle.

In the same time very essential differences:

- (1) the space-time is **four-dimensional** – a ”compactification without compactification” as **alternative to higher dimensions,**
- (2) a natural **consistency with gravity**,
- (3) characteristic parameter of the Kerr strings $a = \hbar/m$ corresponds to **Compton scale of particle physics,**
- (4) **The Kerr-Newman soliton (bag-bubble source) removes contradiction between Quantum theory and Gravity,**
- (5) **Bag deformations** $\Rightarrow$ **circular string** at the border of disk-like source,
- (6) A hint that the Compton region of a **dressed electron forms a bag** - similar to hadronic MIT and SLAC bags,
- (7) **Stringy excitations** create **“zitterbewegung ”** of a pole – pointlike **bare electron**.

N = 2 superstring

The structure of the related with twistors N = 2 superstring is strikingly similar to complex source of Kerr geometry. Both, the bosonic and fermionic parts of the N = 2 superstring work are parallel with the complex source of the complexified 4d Kerr geometry.

THANK YOU FOR ATTENTION!