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Marcinkiewicz Exponents and Their Application in Boundary-Value Problems

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Abstract—We introduce certain new characteristics for non-rectifiable curves which allow to sharpen known solvability conditions for so-called jump boundary-value problems on that curves.

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Introduction. The Riemann boundary-value problem is a classical problem of the complex analysis. Its investigation for smooth and piecewise-smooth contours is one of recognized achievements of Soviet Mathematics (see [1, 2]). A solution to this problem for non-rectifiable curve was obtained thirty years ago [3, 4]. We cite one of main results on the problem, and define to this end certain necessary concepts.

The Hölder condition. Let Γ be a compact set on the complex plane, $0<\nu\leq 1$. A function f defined on Γ satisfies the Hölder condition with the exponent ν if

$$\sup\left\{\frac{|f(t')-f(t'')|}{|t'-t''|^\nu} : t', t'' \in \Gamma,\ t' \neq t''\right\} := h_\nu(f,\Gamma) < \infty,$$

and $H_\nu(\Gamma)$ is the set of all functions satisfying this condition.

Upper metric dimension. We denote by $N(\varepsilon,\Gamma)$ the least number of disks of diameter ε that cover Γ. Then the upper metric dimension of Γ is (see, e.g., [5])

$$\overline{\mathrm{dm}}\,\Gamma := \limsup_{\varepsilon\to 0}\frac{\ln N(\varepsilon,\Gamma)}{-\ln\varepsilon}.$$

The jump problem. Let Γ be a simple closed Jordan curve (generally speaking, non-rectifiable) dividing the complex plane into finite domain D^+ and containing the point at infinity domain D^-. The jump problem is a boundary-value problem on evaluation of holomorphic in $\overline{\mathbb{C}}\setminus\Gamma$ and continuous in $\overline{D^+}$ and in $\overline{D^-}$ function $\Phi(z)$ such that its limits at a point $t\in\Gamma$ from domains D^+ and D^- are related by the equality

$$\Phi^+(t)-\Phi^-(t)=f(t),\quad t\in\Gamma. \tag{1}$$

This problem is a simple and in many ways the most important particular case of the Riemann boundary-value problem.

Now we can formulate the mentioned above result.

Theorem 1 ([3, 4]). *Let Γ be a simple closed Jordan curve (generally speaking, non-rectifiable) on the complex plane and $f\in H_\nu(\Gamma)$. If*

$$\nu > \frac{1}{2}\overline{\mathrm{dm}}\,\Gamma, \tag{2}$$

then the jump problem (1) has a solution.

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