ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2014, Vol. 58, No. 3, pp. 31–46. © Allerton Press, Inc., 2014.
Original Russian Text © S.N. Timergaliev, 2014, published in Izvestiya Vysshikh Uchebnykh Zavedenii. Matematika, 2014, No. 3, pp. 40–56.

On the Existence of Solutions to Geometrically Nonlinear Problems for Shallow Timoshenko-Type Shells with Free Edges

S. N. Timergaliev[1*]

[1]*Naberezhnye Chelny Institute (Branch) of Kazan (Volga Region) Federal University, pr. Mira 68/19, Naberezhnye Chelny, 423810 Russia*

Received September 30, 2012

Abstract—In the paper we investigate the solvability of the boundary-value problems for shallow isotropic elastic shells within the framework of Timoshenko's shear model. The considered problems are nonlinear geometrically and linear physically. The method of studying consists in reducing the initial system of equilibrium equations to one nonlinear differential equation with respect to deflections. In doing so integral representations for the tangential displacements and angles of rotation play a significant role. The representations are deduced by making use of general solutions to the inhomogeneous Cauchy–Riemann equation. The solvability is established by the principle of contracting mappings.

DOI: 10.3103/S1066369X14030049

Keywords and phrases: *Timoshenko-type shell, system of equilibrium equations, boundary-value problem, generalized displacements, generalized problem solution, integral representations, Sobolev spaces, operator, integral equations, holomorphic functions, existence theorem.*

Introduction. Nowadays the solvability of nonlinear problems for thin elastic shells has been studied with enough completeness only within the framework of the Kirchhoff–Love model. For more general models the results concerning the solvability are scanty (see [1–4] and the bibliography therein). The existence theorems for nonlinear problems of shallow Timoshenko-type shells with rigidly clamped edges have been proved in [5, 6]. To this end, a method based on the integral representations for the generalized displacements has been proposed. In [7] this method has been developed for the case of the Timoshenko-type shells with simply supported edges. In the present paper we apply an analogous approach to investigate the solvability of geometrically nonlinear problems within Timoshenko's shear model for shallow shells with free edges.

1. Problem formulation. Concept of generalized solutions. We consider the following model of shallow Timoshenko-type shells.

1) the deformation-displacement relationships ([8], pp. 168–170, 269):

$$\begin{gathered}\gamma_{jj}^0 = w_{j\alpha^j} - k_j w_3 + w_{3\alpha^j}^2/2, \;\; j=1,2, \;\; \gamma_{12}^0 = w_{1\alpha^2} + w_{2\alpha^1} + w_{3\alpha^1}w_{3\alpha^2}, \\ \gamma_{jj}^1 = \psi_{j\alpha^j}, \;\; j=1,2, \;\; \gamma_{12}^1 = \psi_{1\alpha^2} + \psi_{2\alpha^1}, \\ \gamma_{j3}^0 = w_{3\alpha^j} + \psi_j, \;\; j=1,2, \;\; \gamma_{33}^0 = \gamma_{k3}^1 \equiv 0, \;\; k=\overline{1,3},\end{gathered} \tag{1}$$

where γ_{ij}^k $(i,j=\overline{1,3},\, k=0,1)$ are the components of deformation of the shell middle surface S_0, w_i and w_3 are the tangential and normal displacements of the points of S_0, ψ_i $(i=1,2)$ are the rotation angles of normal cross-sections, α^1 and α^2 are the Cartesian coordinates of the points on the plane bounded domain Ω with the boundary Γ, homeomorphic to S_0;

2) defining relationships: $\sigma^{ij} = B^{ijkn}\gamma_{kn}$, $i \le j, k \le n$; $i,j,k,n=\overline{1,3}$; hereinafter summing over repeating Latin indices is from 1 to 3, summing over Greek indices is from 1 to 2, where $\gamma_{kn} = \gamma_{kn}^0 +$

[*]E-mail: Samat_tim@mail.ru.