Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 42, No. 11, pp. 73–78, 1998 *UDC 512.64*

ON A THEOREM BY OSTROVSKIĬ

A.I. Perov

In the present article we suggest a criterion (containing a parameter) of regularity (invertibility) of square matrices. For particular values of the parameter, this criterion turns into known Ostrovskiĭ criteria on nondegeneracy of matrices with constraints upon the maximums of absolute values of non-diagonal elements of either lines, or columns. We shall cite the proof of the regularity criterion, based on the use of fractional degree of a diagonal matrix. We cite metrics in which some matrices turn to be (strict) contractions. On the base of concavity of the second iteration of a certain function, we establish an important inequality between a constant, which stands in the regularity criterion, and the spectral radius of the majorant. In the conditions of the regularity criterion we give estimates on the absolute values of the elements of the inverse matrix. We obtain more fine estimates for the diagonal elements of the inverse matrix. We cite an estimate from below for the absolute value of the determinant of the initial matrix (which gives, in particular, another proof for the suggested regularity criterion).

Let $A = (a_{ij})$ be a complex square $n \times n$-matrix with nonzero diagonal elements

$$a_i \equiv |a_{ii}| > 0, \quad i = 1, \dots, n. \tag{1}$$

Let us compute the following characteristics of non-diagonal elements of the lines and columns:

$$p_i = \max_{j \neq i} |a_{ij}|, \quad q_i = \max_{j \neq i} |a_{ji}|, \quad i = 1, \dots, n \tag{2}$$

(the maximum it taken over all indices $j = 1, \dots, n$, except $j = i$). The following question seems to be of interest: Under what constraints upon a_i, p_i, and q_i the matrix A is regular? We introduce "dimensionless" quantities

$$\varepsilon_i = p_i / a_i, \quad \delta_i = q_i / a_i, \quad i = 1, \dots, n. \tag{3}$$

In what follows we shall assume without special notes that $n \geq 2$ and all the numbers ε_i and δ_i are positive.

Theorem. *Let a complex square $n \times n$-matrix $A = (a_{ij})$ satisfy condition* (1) *and with a certain value of the parameter α, $0 \leq \alpha \leq 1$, the following condition take place*

$$\sigma \equiv \sum_{i=1}^{n} \frac{v_i}{1 + v_i} < 1, \tag{4}$$

where

$$v_i = \varepsilon_i^{1-\alpha} \delta_i^{\alpha}, \quad i = 1, \dots, n. \tag{5}$$