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Solution of Certain Weakly Singular Integral Equations of the First Kind with Indefinite Parameters

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Abstract—In this paper we study a two-dimensional weakly singular integral equation of the first kind with logarithmic kernel. We construct a pair of spaces of the desired elements and the right-hand sides, where we prove the correctness of the problem under consideration and obtain inversion formulas for the integral operator.

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INTRODUCTION

The class of integral equations of the first kind with a logarithmic singularity in the kernel contains widely applied (see, e.g., [1, 2]) equations with indefinite parameters. In this paper we study, as an example, a two-dimensional weakly singular integral equation of the first kind

$$Kx \equiv \frac{1}{\pi^2}\int_{-1}^{1}\int_{-1}^{1}\sqrt{(1-t^2)(1-\tau^2)}\ln|t-s|\ln|\tau-\sigma|x(t,\tau)dt\,d\tau \\ + \alpha\sin(k_1 s)\sin(k_2\sigma) + \beta\cos(k_1 s)\cos(k_2\sigma) + \gamma\sin(k_1 s)\cos(k_2\sigma) \\ + \delta\cos(k_1 s)\sin(k_2\sigma) + V(x;s,\sigma) = f(s,\sigma). \quad (1)$$

Here V is a certain integral operator; $f(s,\sigma)$ is a known function continuous in the domain $[-1,1;-1,1]=[-1,1]^2$; $x(t,\tau)$ is the desired function; α, β, γ, and δ are desired parameters, while k_1 and k_2 are given nonzero parameters such that $J_0(k_i)\neq 0$, $J_1(k_i)\neq 0$ $(i=1,2)$, where $J_r(k)$ is the Bessel function of the rth order.

As usual, integral equations of the first kind are ill-posed. This is connected with complete continuity of integral operators in the known functional spaces. In addition, the questions of the exact or numerical solution of multidimensional integral equations of the first kind are studied in far lesser degree than those of one-dimensional integral equations.

In what follows, using the method and the scheme described in [3, 4], we construct the space $\overline{X}$ of the desired elements and the space Y of the right-hand sides, where Eq. (1) becomes well-posed in the Hadamard sense. We also obtain the inversion formulas for the integral operator with logarithmic kernel.

1. THE SOLVABILITY OF WEAKLY SINGULAR INTEGRAL EQUATIONS WITH INDEFINITE PARAMETERS

Let $X=L_{2\rho}=L_{2\rho}[-1,1]^2$ stand for the space of functions square summable in the Lebesgue sense on $[-1,1]^2$ with the weight $\rho(t,\tau)=\sqrt{(1-t^2)(1-\tau^2)}$ and with the usual norm

$$\|\varphi\|_X = \|\varphi\|_{2\rho} = \left(\int_{-1}^{1}\int_{-1}^{1}\rho(t,\tau)|\varphi(t,\tau)|^2dt\,d\tau\right)^{\frac{1}{2}}, \quad \varphi\in X.$$

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