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A Nonlocal Boundary-Value Problem with Conormal Derivative for a Mixed-Type Equation with Two Inner Degeneration Lines and Various Orders of Degeneracy

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Abstract—We prove the unique solvability of the boundary value problem with conormal derivative for a mixed-type equation with two inner degeneration lines and with various orders of degeneracy.

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In papers [1–6] one considers the Tricomi problem and its analogs for a mixed-type equation with two degeneration lines and with various orders of degeneracy on a part of the boundary of a bounded or unbounded domain.

Papers [7–13] are dedicated to boundary-value problems with Poincaré conditions or with a conormal derivative for Tricomi and Lavrent'ev–Bitsadze equations and more general ones.

However, local and nonlocal boundary-value problems with conormal derivative for mixed-type elliptic-hyperbolic equations with two inner degeneration lines and various orders of degeneracy are not studied yet.

In this paper we prove the unique solvability of a nonlocal boundary-value problem with conormal derivative for a mixed-type elliptic-hyperbolic equation with two inner degeneration lines and various orders of degeneracy.

Consider the equation

$$\operatorname{sign} y\,|y|^m\,u_{xx} + \operatorname{sign} x\,|x|^n\,u_{yy} = 0, \tag{1}$$

where m and n are piecewise constant functions of x and y such that

$$m(n)(x,y) = \begin{cases} m(n) & \text{with } x>0,\ y>0;\\ m_1(n) & \text{with } x>0,\ y<0;\\ m(n_1) & \text{with } x<0,\ y>0. \end{cases} \tag{2}$$

Denote by Ω a finite simply connected domain in the plane (x,y) such that it is bounded by a curve σ with endpoints at $A\,(h_1,0)$ and $B\,(0,h_2)$ if $x>0$ and $y>0$, while for $x>0$, $y<0$ and $x<0$, $y>0$ it is bounded by characteristics

$$OC:\ \frac{1}{q}x^q - \frac{1}{p_1}(-y)^{p_1} = 0, \quad AC:\ \frac{1}{q}x^q + \frac{1}{p_1}(-y)^{p_1} = 1$$

and

$$OD:\ \frac{1}{q_1}(-x)^{q_1} - \frac{1}{p}y^p = 0, \quad BD:\ \frac{1}{q_1}(-x)^{q_1} + \frac{1}{p}y^p = 1$$

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