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A Note on Δ_2^0-Spectra of Linear Orderings and Degree Spectra of the Successor Relation

A. N. Frolov[1*]
(Submitted by M.M. Arslanov)

[1] *Kazan (Volga Region) Federal University, ul. Kremlyovskaya 18, Kazan, 420008 Russia*
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Abstract—In this paper we construct linear orderings whose Δ_2^0-spectra coincide with classes of all high$_0$ and high$_1$ degrees, respectively. We also prove that there exists a computable linear ordering such that its degree spectrum of the successor relation coincides with a fixed nonempty class of degrees which represents a Σ_1^0-spectrum of some $\emptyset'$-computable linear ordering.

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1. INTRODUCTION

The research area of this paper is the theory of computable linear orderings. A linear ordering $(\mathbb{N}, <_L)$ is called *computable*, X*-computable* (and other), if the order relation $<_L$ is computable, X-computable (and other), respectively. The main terminology of the computability theory, that we use in this work, can be found in the book by R. Soare [1], and the main terminology of the theory of linear orderings can be found in the book by J. Rosenstein [2]. In the latter book one can find the main research directions of the theory of computable linear orderings. The paper [3] by R. Downey is one of the most recent review papers on the computable structure theory and, in particular, the theory of computable linear orderings.

One of fundamental problems of the theory of computable linear orderings is to describe spectra of linear orderings [3]. The *spectrum* of a linear ordering L is the class $\mathrm{Spec}(L) = \{\deg_T(\widetilde{L}) \mid \widetilde{L} \cong L\}$.

The number of examples of spectra of linear orderings known by now is not so large. Such examples are given in the recent paper [4] by the author and V. Harizanova, I. Kalimullin, O. Kudinov, and R. Miller. In connection with the study of spectra of linear orderings, R. Miller [5] notes the importance of the study of bounded spectra and, in particular, of Δ_2^0-spectra. The Δ_2^0-*spectrum* of a linear ordering L is the class of Turing degrees $\mathrm{Spec}^{\Delta_2^0}(L) = \mathrm{Spec}(L) \cap \Delta_2^0$. In other words, $\mathrm{Spec}^{\Delta_2^0}(L) = \{\deg_T(\widetilde{L}) \in \Delta_2^0 \mid \widetilde{L} \cong L\}$.

In the same paper, R. Miller constructs a linear ordering whose Δ_2^0-spectrum contains exactly all nonzero Δ_2^0-degrees. As was noted later, the Miller linear ordering has representations in all hyperimmune degrees. The study of this ordering is not finished yet.

In this paper we give some new examples of Δ_2^0-spectra of linear orderings. Since the Δ_2^0-spectrum of a linear ordering is closed upwards in Δ_2^0-degrees, it is natural to consider classes of Δ_2^0-degrees which are also closed upwards. Namely, similarly to [4], we consider classes of Δ_2^0-degrees, which are not low$_n$, and classes of high$_n$ Δ_2^0-degrees.

If $X^{(n)} \leq_T \emptyset^{(n)}$, then the set X and the degree $\deg_T(X)$ are called *low*$_n$. We denote the class of all degrees different from low$_n$ ones by $\mathbf{NonLow_n}$. Note that there exists only one low$_0$ degree, namely, the degree of the empty set $\deg_T(\emptyset) = \mathbf{0}$.

If $\emptyset^{(n+1)} \leq_T X^{(n)}$, then the set X and the degree $\deg_T(X)$ are called *high*$_n$. We denote the class of all high$_n$ degrees by $\mathbf{High_n}$. Note that there exists only one high$_0$ degree, namely, the degree $\mathbf{0}'$.

*E-mail: Andrey.Frolov@mail.ru.