Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 49, No. 5, pp.1–10, 2005 *UDC 514.752*

GEOMETRY OF SUBMANIFOLDS WITH STRUCTURE OF DOUBLE BUNDLE IN PSEUDO-EUCLIDEAN RASHEVSKII SPACE

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1. Introduction

The Rashevskii space is an even-dimensional pseudo-Riemannian manifold with metric of half-index. This space was introduced in [1] as an example of a Riemannian space with interesting properties, and also as a hyperbolic analog of so-called A-spaces of elliptic type introduced in [2]. Later this space was investigated in details in [3], where parabolic A-spaces were introduced, however no possible applications were studied.

P.K. Rashevskii investigated an invariant scalar field $U(x^1, \dots, x^n, y_1, \dots, y_n)$ whose matrix of second order partial derivatives

$$\det\left(\frac{\partial^2 U}{\partial x^j \partial y_i}\right) \neq 0, \quad i, j = 1, 2, \dots, n,$$

is nonsingular, and introduced a pseudo-Riemannian metric of index n on a $2n$-dimensional manifold M with local coordinates $x^1, \dots, x^n$, $y_1, \dots, y_n$, and the corresponding pseudo-Riemannian connection. This manifold has the following properties [1].

1. The scalar field $U(x^1, \dots, x^n, y_1, \dots, y_n)$ generating the structure of pseudo-Riemannian space on M is determined up to two arbitrary summands

$$U(x^i, y_j) \to U(x^i, y_j) + U_1(x^i) + U_2(y_j).$$

2. The manifold M consists of two families of n-dimensional fibers. Any point of M lies in one and only one fiber of each family of fibers, and fibers from different families meet each other at most at one point.
3. Fibers of both families are isotropic.
4. Fibers of each family are absolutely parallel, this means that the parallel translation along any curve maps a vector tangent to a surface of a family to a vector which is also tangent to a surface of the same family.

From the last two properties it follows that both families of fibers are totally geodesic in M.

In [4] it was discovered that these spaces are naturally related to some classes of n-tuple integrals depending on n parameters: if the matrix of second order partial derivatives of the logarithm of integrand is not degenerate, then this integral generates a structure of $2n$-dimensional Rashevskii space on the $2n$-dimensional manifold of the double fiber bundle of variables and parameters. Moreover, this structure does not change if we multiply the integrand by an arbitrary smooth function depending only on variables (or parameters).