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The Averaging Method and the Asymptotic Behavior of Solutions to Differential Inclusions

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Abstract—We prove one version of the first Bogolyubov theorem for differential inclusions with multivalued mappings that satisfy certain one-sided constraints. We study the dependence of solutions to differential inclusions on the parameters.

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The main result of this paper is the substantiation of one variant of the first Bogolyubov theorem [1] for differential inclusions.

Let us use the following main denotations: $\|x; E\| = \|x\|_E$ is the norm of the element x in the B-space E, E^* is the space conjugate to E, $\langle x, x^*\rangle$ is the value of the functional $x^* \in E^*$ on the element $x \in E$, $\sigma(E, E^*)$ is the weak topology on E generated by the form $\langle \cdot, \cdot\rangle$, $s(x^*, M) = \sup\{\langle x, x^*\rangle, x \in M\}$ is the support function of the set $M \subset E$; $d_E(x, M) = \inf\{\|x - v; E\|, v \in M\}$ is the distance from x to the set M; $\|M\|_E = \sup\{\|v\|_E, v \in M\}$, $\theta_E(M_1, M_2) = \sup\{d_E(x, M_2), x \in M_1\}$ is the deviation of the set $M_1 \subset E$ from that $M_2 \subset E$. Symbols $Kv(E)$ ($\mathfrak{Kv}(E)$) denote the set of convex compact (compact in the topology $\sigma(E, E^*)$) subsets of the space E. All Banach spaces are considered over the field $\mathbb{R}$ of real numbers, $\mathbb{R}_+ = [0, \infty)$, $\mathbb{N}$ is the set of natural numbers, $\mathbb{N}_0 = \{0\} \cup \mathbb{N}$.

A multivalued mapping (m. mapping) F of a set M_1 to that M_2 is an operator that transforms an element v from M_1 to a nonempty set $F(v) \subset M_2$; if $K \subset M_1$, then $F(K) = \cup_{v\in K} F(v)$ is the range space of the mapping F on the set K. One calls an m. mapping $F : M_1 \to M_2$ (M_1 and M_2 are topological spaces) upper semicontinuous on M_1 if for any open set $V \subset M_2$ the set $\{x \mid x \in M_1, F(x) \subset V\}$ is open. If images of an m. mapping $F : M \to E$ (M is a topological space and E is a Banach space) belong to the class $Kv(E)$ ($\mathfrak{Kv}(E)$), then we write it as follows: $F : M \to Kv(E)$ (respectively, $F : M \to \mathfrak{Kv}(E)$). If the space E is considered with the weak topology, then the mapping $F : M \to \mathfrak{Kv}(E)$ is upper semicontinuous if and only if for any element x^* from E^* the function $v \to s(x^*, F(v))$ is upper semicontinuous on M ([2], pp. 126–127). Without additional explanations we use the terminology of the theory of measurable m. mappings ([3–6]).

Further $T = [0, l]$ ($0 < l < \infty$), H is a finite-dimensional Euclidean space with the scalar product $(\cdot, \cdot)$ and the norm $|x| = \sqrt{(x, x)}$, $L^p(T, H)$ and $C(T, H)$ are the spaces of Lebesgue measurable (continuous) on the segment T functions with values in H (as usual, $1 \le p \le \infty$, equivalent functions are identified, norms in $L^p(T, H)$ and $C(T, H)$ are defined in the standard way); $\Pi(T, H)$ is the family of functions $u : T \to H$ that are constant on segments that form a finite partition of the segment T.

Proposition 1 ([7], P. 238). *Let $M \subset L^1(T, H)$. The following assertions are equivalent*:

a) *from any sequence $(v_n)_{n\in\mathbb{N}}$ in M one can extract a subsequence that converges in $\sigma(L^1, L^\infty)$*;

b) *the set M is uniformly summable, i.e., $\|1_\Delta z; L^1(T)\| \le \Psi(\operatorname{mes}\Delta)$ for any z from M and measurable sets $\Delta \subset T$ (here 1_Δ and $\operatorname{mes}\Delta$ are the characteristic function and the Lebesgue measure of the set Δ, respectively, $\Psi(\xi) \to 0$ as $\xi \to 0$)*;

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