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Construction of Almost Periodic Solutions to One Quasilinear System with Two Delays

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Abstract—In this paper we construct an almost periodic solution to a quasilinear system with constant and linear delays. We prove the asymptotic stability of this solution.

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Let us consider a nonhomogeneous quasilinear system with two delays

$$dx(t)/dt = Ax(t) + B_1x(t-\tau) + B_2x(\mu t) + f(t) + \nu F(t, x(t), x(t-\tau), x(\mu t)),$$
$$t \geq t_0 \geq t_* > 0,\ \ \tau = \text{const},\ \ \tau > 0,\ \ \mu = \text{const},\ \ 0 < \mu < 1. \tag{1}$$

Here A, B_i $(i = 1, 2)$ are constant matrices of dimension $m \times m$; $f(t)$ is an m-dimensional almost periodic vector function of time, it is k times differentiable ($f(t) \subset \sqcap^{(k)}$); $x(t)$ is an m-dimensional vector function of time (the argument) t. The delay τ is constant, while the delay $(1-\mu)t$ is a linear function of time t. The nonlinear m-dimensional vector function $F(t, u, v, w)$ is also almost periodic with respect to t and differentiable sufficiently many times, in a closed bounded neighborhood of variables (u, v, w) if satisfies the conditions

$$F(t, 0, 0, 0) = 0,\ \ \|F(t, u_1, v_1, w_1) - F(t, u_2, v_2, w_2)\| \leq L[\|u_1 - u_2\| + \|v_1 - v_2\| + \|w_1 - w_2\|],$$
$$L = \text{const},\ \ L > 0,\ \ \nu = \text{const},\ \ \nu > 0; \tag{2}$$

$\|u\| = \max_j |u^j|$, u^j are components of the vector u, $j = 1, 2, \ldots, m$. The parameter ν is sufficiently small. We define the norm of a matrix in accordance with the norm of a vector.

Systems that contain constant and linear delays represent a further generalization of systems of differential equations with only one (linear) delay ([1], P. 150). Based on asymptotic properties of linear homogeneous systems studied by us earlier [2], we construct almost periodic solutions to system (1) with $t > 0$, because with negative values of the argument the system is not a system with aftereffect.

Let $\overline{T}_\varepsilon$ be a general ε-almost period of the vector function $f(t)$ and the family $F(t, u, v, w)$; $\|u\| + \|v\| + \|w\| \leq b$; here b is a positive constant. Let us first consider a generating system, i.e., system (1) with $\nu = 0$,

$$d\widehat{x}(t)/dt = A\widehat{x}(t) + B_1\widehat{x}(t-\tau) + B_2\widehat{x}(\mu t) + f(t),\ \ t \geq t_0 \geq t_* > 0. \tag{3}$$

Together with this system we consider the homogeneous system

$$dy(t)/dt = Ay(t) + B_1y(t-\tau) + B_2y(\mu t),\ \ t \geq t_0 \geq t_* > 0, \tag{4}$$

and the "truncated" homogeneous system

$$dz(t)/dt = Az(t) + B_1z(t-\tau),\ \ t \geq 0. \tag{5}$$

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