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Estimate of the Order of Growth of a Class of Entire Functions on the Real Axis

E. Mukhamadiev* **and A. N. Naimov****

Vologda State University
ul. Lenina 15, Vologda, 160000 Russia

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Abstract—For a class of entire functions we study the problem of estimation of the order of growth of functions on the real axis. This problem is important for the justification of the integral representation of bounded solutions to certain partial differential equations considered in other papers of the authors. In order to obtain an estimate of the order of growth of a function on the real axis, we use the method of differential equations. The method is based, on one hand, on the construction of a system of first-order ordinary differential equations whose solution is a vector function of traces of function and its derivatives on the real axis. On the other hand, under the respective change of variables in the system of equations, we obtain an estimate of the solution to the system of equations for a large positive values of the argument. The obtained estimate is non-trivial and shows the way a complex parameter of a power series affects the order of growth of a function.

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INTRODUCTION

We consider a class of entire functions of the form

$$F_{b,m_1,\dots,m_n}(z)=\sum_{k=0}^{\infty}\frac{(b\cdot z)^k}{(m_1(k+1)-1)!\cdot\ldots\cdot(m_n(k+1)-1)!}. \tag{1}$$

Here $z=t+i\sigma$ is a complex variable, b is a nonzero complex number, $n\geq 2$, $m_1,\dots,m_n$ are natural numbers. For function $F_{b,m_1,\dots,m_n}(z)$, we study the question of estimation of its order of growth on the real axis, namely, the estimation of the absolute value $|F_{b,m_1,\dots,m_n}(t)|$ of function $F_{b,m_1,\dots,m_n}(t)$, $t\in(-\infty,+\infty)$, and the absolute values of its derivatives $\left|F^{(l)}_{b,m_1,\dots,m_n}(t)\right|$, $l=1,\dots,m_1+\dots+m_n-1$, for a large positive real-valued t. This problem is important for the justification of the integral representation of bounded solutions to certain partial differential equations considered in [1, 2].

Notice that the absolute value of function $F_{b,m_1,\dots,m_n}(t)$ and the absolute values of its derivatives can be roughly estimated by $|b|$. Still, such estimates are insufficient for the complete justification of the integral representations of bounded solutions obtained in the above mentioned articles. In order to obtain the estimate of the order of growth of function $F_{b,m_1,\dots,m_n}(t)$, we need not only $|b|$, but also we need to consider the argument $\arg(b)$ of a complex number b.

In this paper, in order to obtain an estimate of the order of growth of function $F_{b,m_1,\dots,m_n}(t)$, we use the method of differential equations. The method is based, on one hand, on the construction of a system of first-order ordinary differential equations with the solution being a vector-function $(F_{b,m_1,\dots,m_n}(t),F'_{b,m_1,\dots,m_n}(t),\dots,F^{(m-1)}_{b,m_1,\dots,m_n}(t))^\top$, where $m=m_1+\dots+m_n$. On the other hand,

*E-mail: emuhamadiev@rambler.ru.
**E-mail: nan67@rambler.ru.