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NEW VERSION OF THE COLLOCATION METHOD FOR A CLASS OF INTEGRODIFFERENTIAL EQUATIONS

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We consider a linear integrodifferential equation (IDE) of the form

$$Ax \equiv x(t)\prod_{j=1}^{q}(t-t_j)^{m_j}+\sum_{j=0}^{p}\int_{-1}^{1}K_j(t,s)x^{(j)}(s)ds=y(t), \quad -1\le t\le 1, \tag{1}$$

where $t_j\in(-1,1)$, $m_j\in\mathbf{N}$, $j=\overline{1,q}$; K_j, $j=\overline{0,p}$, and y are known smooth functions, while x is the desired function. Investigation of such equations is of interest from both the applications and theory (in particular, IDE (1) is a generalization of a series of classes of integrodifferential equations of Fredholm type). A series of important problems of the transport theory, theory of elasticity, scattering theory (see, e.g., [1] and bibliography therein; [2]), theory of mixed equations (see [3]), and also theory of some loaded IDE (see [4]) lead to problems of that kind. Since the IDE under investigation can be solved in exact form only in very rare cases, the elaboration of effective technique for their approximate solving with the corresponding theoretical substantiation turns to be significantly actual. A series of results in this direction were obtained in [5]–[16], where special direct methods for solving integral equations of the third kind (i.e., IDE (1) for $p=0$) and second kind (IDE (1) for $m_j=0$, $j=\overline{1,q}$, $K_j=0$, $j=\overline{1,p}$) were suggested and justified. In [17], on the basis of standard polynomials, a direct projection method for solving IDE (1) in the class of generalized functions was constructed.

In this article we suggest new versions of collocation method, which are adjusted for solving IDE (1). The principal attention is devoted to the justification of methods under investigation in the sense of Chapter I in monograph [18]. Namely, we prove theorems of existence and uniqueness of a solution of the corresponding approximate equation, establish estimates of error of the approximate solution, and prove the convergence of approximate solution to the exact solution in a certain space of generalized functions. We also investigate stability and conditionality of the approximating equations.

1. *Basic spaces.* Let $C\equiv C(I)$ be a space of continuous on $I\equiv[-1,1]$ functions with usual max-norm and $m\in\mathbf{N}$. Following [19], we say that a function $f\in C$ belongs to the class $C\{m;0\}\equiv C_0^{\{m\}}(I)$ if at the point $t=0$ the Taylor derivative $f^{\{m\}}(0)$ of order m exists (we evidently assume that $C\{0;0\}\equiv C$). By the norm we have

$$\|f\|_{C\{m;0\}}\equiv\|Tf\|_C+\sum_{i=0}^{m-1}|f^{\{i\}}(0)|,$$

where

$$Tf\equiv\Big[f(t)-\sum_{i=0}^{m-1}f^{\{i\}}(0)t^i/i!\Big]t^{-m}\equiv\mathcal{F}(t)\in C, \quad \mathcal{F}(0)\equiv\lim_{t\to 0}\mathcal{F}(t),$$