

Injective Objects in the Category of Stratified Spaces

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Abstract—The paper deals with the category of stratified spaces. For this category we establish the existence of the absolute extensors and study their connection with the absolute extensors in the category of filtered spaces. To this end, we introduce and investigate the absolute extensors reflecting the properties of both the filtered category and the stratified category. We determine the contractibility property as well as the local contractibility property for the stratified absolute extensors. It is shown that the family of strata for a stratified absolute neighbourhood extensor is equicontinuously locally extendable.

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Introduction. Spaces with countable filtrations consisting of closed subsets are of great interest in modern topology. One of the fruitful approaches to the research in those spaces is the study of extensor properties in the category of filtered spaces (or the \mathcal{N} -category). The objects in this category are the filtered metric spaces, and its morphisms are the filtered maps, i.e., the maps preserving filtrations. Arising from that approach, the theory of absolute extensors and retracts in the filtered category develops and successfully supplements Milnor's classical results about the cell complexes and the properly filtered spaces ([1], P. 255; [2], P. 405). Fundamentals of this theory are contained in the work [3].

However, it is impossible in principal to track topological and homotopic properties of strata in the filtered spaces within the framework of the filtered category (although it is very desirable in many situations). It is connected with the fact that the filtered maps retaining the elements of a filtration as invariants do not preserve the strata. It follows from this fact, for example, that any injective object in the \mathcal{N} -category (an \mathcal{N} -AE-space) is homotopically equivalent to a point. But nevertheless there exist \mathcal{N} -AE-spaces with homotopically nontrivial strata. Therefore, there is a need for a finer theory which gives an information about strata and describes a situation adequately.

A way out of the situation is possible if one introduces a new category (the \mathcal{S} -category) whose objects are the filtered metric spaces as before, and whose morphisms are the maps preserving the strata (the stratified maps). In that case, the restriction of the homotopical equivalence in this category to any stratum is the homotopical equivalence, and any stratum of an injective object of the \mathcal{S} -category is an absolute retract. However, there is no a priori answer to the question whether the collection of such injective objects is large. It is also not clear whether properties of those objects are good enough.

The main result in the paper is the presentation of a sufficient collection of the injective objects in the \mathcal{S} -category. In fact, we establish a stronger result (Theorem 2). Namely, we show that not only a stratified map but any filtered map taking its values in an \mathcal{S} -AE-space admits a stratified extension. As an immediate consequence of this theorem, we obtain the basic properties of the stratified absolute extensors. In particular, we establish the connection between the \mathcal{N} -AE-spaces and the \mathcal{S} -AE-spaces (Theorem 4). We also study other properties of the \mathcal{S} -AE-spaces.

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