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THE METHOD OF CONFIDENCE NEIGHBORHOODS FOR MINIMIZATION OF CODIFFERENTIABLE FUNCTIONS

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Introduction

In this paper, we propose one algorithm for finding a local minimum of a nonsmooth nonconvex function. We assume that the objective function is codifferentiable (see [1], [2]). This problem has many applications but there are few algorithms for solving it. Our algorithm belongs to the class of confidence neighborhoods techniques [3], [4], which were developed in [5] and [6] for minimization of sum of squares. One of the main advantages of the confidence neighborhoods methods is their global convergence. It is well known that using the confidence neighborhoods may be better than the directional minimization.

The idea of such methods is simple. We construct a function approximating the objective one and minimize it subject to constraints imposed on the step. These constraints guarantee that the next iteration point belongs to the confidence neighborhood. If the objective function decreases sufficiently well, we dilate the confidence neighborhood and move to the next iteration point. Otherwise, we stay at the same iteration point and contract the confidence neighborhood. The auxiliary problem, which implies the minimization of the approximating function, can be solved approximately using the dual estimates.

1. Problem definition

In this paper, we solve the unconstrained minimization problem

$$\min\{f(x) \mid x \in \mathbb{R}^n\}.$$

We assume that the objective function $f(x)$ is bounded below. Let the function $m(s)$ approximate $f(x+s) - f(x)$ locally in some neighborhood of x, where s is a vector with a small norm. We call this function a model. In a differentiable case, we set

$$m(s) = \langle f'(x), s\rangle.$$

For a doubly differentiable function one may use the quadratic model [3]

$$m(s) = \langle f'(x), s\rangle + \langle f''(x)s, s\rangle.$$

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