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AFFINE SYMMETRIES OF QUASIGEODESIC FLOWS

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In the present paper, using geometric (geodesic, spray) simulation, we study affine mobility of quadratic quasigeodesic flows (QF).

In [1] É. Cartan introduced the notion of projectively connected space and demonstrated that the geodesic lines of a projectively connected space locally coincide with the integral curves of a special QF, which are polynomials of third degree in velocities. Also, in this paper he posed the problem to generalize this theory in such a way that integral curves of any QF could be considered as geodesic lines, and solved this problem for the simplest, in his opinion, two-dimensional case, i. e. for the one-dimensional QF. In [2] É. Cartan's results on geodesic simulation were extended to the case of submersion with many-dimensional fibres, and in [3]-[6] É. Cartan's problem on geodesic simulation was completely solved.

I. BASIC NOTIONS

Let M be an $(n-1)$-dimensional manifold, $f = (M, f)$ be a QF on M (a second order differential equation on M) which is written with respect to an arbitrary chart as

$$\frac{d^2x^i}{dt^2} = f^i\left(x^j, t, \frac{dx^j}{dt}\right),$$

where $i, j = 1, \dots, n-1$. In what follows we assume that all objects are of sufficiently large differentiability class.

§ 1. Infinitesimal point symmetries of quasigeodesic flow

For an arbitrary QF (M, f), in [3]–[6] there was constructed a geodesic (spray) model such that the geodesic lines of the standard generalized connection $\overline{\Gamma}$ of f in the event space $\overline{M} = M \times R$ coincide with the integral curves of f.

S. Lie defined the infinitesimal point transformations of differential equations ([7]–[9]). For second order ordinary differential equations or, what is the same, for quasigeodesic flows, this definition can be formulated in the following way.

Definition 1. A vector field X on the event space $\overline{M} = M \times R$ of a QF (M, f) is called an infinitesimal point symmetry (a point transformation) of (M, f) if the one-parameter group of local diffeomorphisms determined by X consists of point mappings which preserve the integral curves of the restrictions of (M, f) to domains of these mappings.

On the event space $\overline{M}$ of a QF (M, f) there are defined the connection $\overline{\Gamma}$ and the distribution $dx^n = dt = 0$. In this connection, in [4]–[6] the following four types of point symmetries of QF are introduced:

1) projective quasisymmetries of QF (PQS);
2) affine quasisymmetries of QF (AQS);