Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 42, No. 2, pp.44–50, 1998 *UDC 517.958 : 539.3*

ASYMPTOTIC SOLUTIONS OF SYSTEM OF EQUATIONS FOR SLOPING SHELLS IN THE FORM OF TWO-DIMENSIONAL WAVE PACKETS

G.I. Mikhasyov

In [1], [2] by V.P. Maslov's method (see [3]) complex WKB-solutions of linear equations of motion of thin shells were constructed in the form of one-dimensional wave packets which run over the surface of a shell. In [4] there was proposed a new idea for construction of solutions concentrated in the vicinity of closed lines. The idea (see [5]) of a passage to a moving system of coordinates, which is connected to the wave packet center, as well as the concrete form (see [6]) of the function of eikonal in the anzats of WKB-series, were put into its base.

The objective of the present article is development of the method suggested in [4] to the case of two-dimensional wave packets. We suggest a WKB-procedure for constructing a formal asymptotic solution of the Cauchy problem for the equation of sloping shells, the solution being concentrated in the vicinity of moving points. It is assumed that the thickness of the shell and physical characteristics of the material vary. We give an example of possible wave forms of the motion of a thin spherical panel with varying Young module and material density.

1. Statement of the problem

Assuming a large variation of the waves in both directions, we use the system of equations of sloping shells, written in dimensionless form (see [7])

$$\begin{aligned} \varepsilon^2\{\Delta d\Delta w - K[(1-\nu)d, w]\} + \Delta_k\Phi + m\partial^2 w/\partial t^2 = 0, \\ \varepsilon^2\{\Delta g\Delta\Phi - K[(1+\nu)g, \Phi]\} - \Delta_k w = 0, \end{aligned} \tag{1}$$

where

$$\begin{gathered} \Delta z = \frac{1}{A_1A_2}\sum_i \frac{\partial}{\partial\alpha_i}\Big(\frac{A_j}{A_i}\frac{\partial z}{\partial\alpha_i}\Big), \quad \Delta_k z = \frac{1}{A_1A_2}\sum_i \frac{\partial}{\partial\alpha_i}\Big(\frac{A_j}{A_iR_j}\frac{\partial z}{\partial\alpha_i}\Big), \\ K[\Psi, z] = \frac{1}{A_1A_2}\sum_i \frac{\partial}{\partial\alpha_i}\Big\{\frac{1}{A_i}\frac{\partial\Psi}{\partial\alpha_j}\Big[\frac{\partial}{\partial\alpha_j}\Big(\frac{1}{A_j}\frac{\partial z}{\partial\alpha_j}\Big) + \frac{1}{A_i^2}\frac{\partial z}{\partial\alpha_i}\frac{\partial A_j}{\partial\alpha_i}\Big] - \\ -\frac{1}{2}\frac{\partial\Psi}{\partial\alpha_j}\sum_k \frac{A_l}{A_k}\frac{\partial}{\partial\alpha_k}\Big(\frac{1}{A_l^2}\frac{\partial z}{\partial\alpha_l}\Big)\Big\}, \quad i,j,k,l = 1,2, \quad l \neq k, \\ d(\alpha_k) = \frac{Eh^3}{1-\nu^2}, \quad g(\alpha_k) = \frac{1}{Eh}, \quad m(\alpha_k) = \rho h. \end{gathered} \tag{2}$$

In the capacity of curvilinear coordinates α_1, α_2 on the middle surface there are taken the curvature lines, so the first quadratic form of the surface takes the form $d\sigma^2 = R_0^2(A_1^2\, d\alpha_1^2 + A_2^2\, d\alpha_2^2)$, where