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The Unique Solvability of Certain Multiplicative-Convolution Equations

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Abstract—In the class of distributions of slow (moderate) growth we consider a class of equations with operations of convolution and multiplication on the real axis. This class contains convolution equations, in particular, ordinary differential equations with constant coefficients, equations in finite differences, functional differential equations with constant coefficients and shifts, and pair differential equations. By virtue of the analytic representation theory for distributions of moderate growth (the Hilbert or Cauchy transform) the class of equations under consideration is equivalent to the class of boundary value problems of the Riemann type, where an equation corresponds to a boundary value condition in the sense of distributions of moderate growth. As a research technique we use the Fourier transform, the generalized Fourier transform (the Carleman—Fourier transform), and the theory of convolution equations in the space of distributions of moderate growth.

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In the space $\mathcal{S}'$ of distributions of slow (moderate) growth we consider the equation

$$A_+[F * \Phi]_+ - B_-[G * \Phi]_- = W, , \tag{I}$$

where A_+ and B_- are, respectively, given distributions from multiplicative algebras $\mathcal{F}(\mathcal{S}'_+)$ and $\mathcal{F}(\mathcal{S}'_-)$ isomorphic to convolution algebras $\mathcal{S}'_+$ and $\mathcal{S}'_-$ ([1], P. 122) of distributions from $\mathcal{S}'$ with supports in $\mathbb{R}_+ := [0; +\infty)$ and $\mathbb{R}_- := (-\infty; 0]$, correspondingly, under the Fourier transform $\mathcal{F}$ based on the formula (ibid., P. 126)

$$\langle \mathcal{F}T, \varphi\rangle := \langle T, \mathcal{F}\varphi\rangle \ \ \forall\varphi \in \mathcal{S}, \ \ \forall T \in \mathcal{S}',$$

where $(\mathcal{F}\varphi)(\xi) := \int\limits_{\mathbb{R}} e^{2\pi i x\xi}\varphi(x)dx \quad \forall\xi \in \mathbb{R}$; the distributions F and G belong to the space Θ'_c of convolutors for the space $\mathcal{S}'$; $[F * \Phi]_+$ and $[G * \Phi]_-$ are boundary values (in the sense of $\mathcal{S}'$) of analytical representations $\big(\widehat{F * \Phi}\big)(z)$ and $\big(\widehat{G * \Phi}\big)(z)$ for $\operatorname{Im} z \to +0$ and $\operatorname{Im} z \to -0$, correspondingly; W and Φ are, respectively, the given and desired distributions from $\mathcal{S}'$.

The convolution $F * \Phi$ is defined by the formula

$$\langle F * \Phi, \varphi\rangle := \langle \Phi, \check{F} * \varphi\rangle \ \ \forall\varphi \in \mathcal{S},$$

where $\langle \check{F}, \varphi\rangle := \langle F, \check{\varphi}\rangle$, $\check{\varphi}(x) := \varphi(-x)$.

The multiplication of elements A_+ and T_+ of the algebra $\mathcal{F}(\mathcal{S}'_+)$ is defined by the formula

$$A_+T_+ := \mathcal{F}\{\overline{\mathcal{F}}A_+ * \overline{\mathcal{F}}T_+\},$$

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