Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 49, No. 5, pp. 64–66, 2005 *UDC 514.763*

CURVILINEAR THREE-WEBS ADMITTING ONE-PARAMETER FAMILY OF AUTOMORPHISMS

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Introduction. The study of geometry of three-webs was initiated by W. Blaschke and his pupils, participants of Hamburg geometrical seminar, at the end of twenties of the XX century. Since that time the great amount of literature on this theme has been written (see, e. g., review [1]). One can notice that a majority of the latest works is devoted to the multidimensional three-webs. In particular, multidimensional three-webs admitting various families of automorphisms have been intensively investigated (see [2] and the bibliography in [1], [2]). However, special classes of three-webs of minimal dimensions, namely the curvilinear three-webs on the plane, have not been adequately studied. In particular, the three-webs admitting one-dimensional family of automorphisms remain little investigated. (Note that a curvilinear three-web admitting a two-parameter family of automorphisms is regular, this means that this three-web is equivalent to a three-web generated by three families of parallel straight lines.) Along these lines we can mention only one paper [3], where the authors find a first order differential equation determining this class of webs. In the present paper we completely integrate this differential equation (by another method) and find the general finite equation for a curvilinear three-web admitting one-parameter family of automorphisms.

1. Following [2], we define the foliations of a curvilinear three-web W by the Pfaff equations

$$\omega_1 = 0, \quad \omega_2 = 0, \quad \omega_1 + \omega_2 = 0. \tag{1}$$

The basic forms ω_1 and ω_2 satisfy the structure equations

$$d\omega_1 = \omega_1 \wedge \omega, \quad d\omega_2 = \omega_2 \wedge \omega \tag{2}$$

and

$$d\omega = b\omega_1 \wedge \omega_2. \tag{3}$$

From the last equation, by exterior differentiation we obtain the differential equations for the covariant derivatives of the curvature b:

$$\begin{gathered} db - 2b\omega = b_1\omega_1 + b_2\omega_2, \\ db_1 - 3b_1\omega = b_{11}\omega_1 + b_{12}\omega_2, \quad db_2 - 3b_2\omega = b_{21}\omega_1 + b_{22}\omega_2, \\ db_{11} - 4b_{11}\omega = b_{112}\omega_1 + b_{112}\omega_2 \end{gathered} \tag{4}$$

etc. The covariant derivatives are related to each other, e. g.,

$$b_{12} - b_{21} = 2b^2.$$