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Generalized Solutions to Boundary-Value Problems for Quasilinear Elliptic Equations on Noncompact Riemannian Manifolds

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Abstract—In the present work we develop approximation approach to evaluation of solutions to boundary-value problems for quasilinear equations of the elliptic type on arbitrary noncompact Riemannian manifolds. Our technique essentially bases on an approach from the papers of E. A. Mazepa and S. A. Korol'kov connected with introduction of equivalency classes of functions and representations. On the other hand, it generalizes the method of building of generalized solution to the Dirichlet problem for linear elliptic Laplace–Beltrami and Schrödinger equations in bounded domains in $\mathbb{R}^n$, which is described in details in the works of M. V. Keldysh and E. M. Landis.

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INTRODUCTION

At present we know a number of ways to definition of the concept of generalized solution for elliptic equations. One of these ways is based on introduction of appropriate Hilbert space; it enables us to define the action of elliptic operator on a class of functions, which is essentially more extensive than C^2 (see [1], pp. 170–171).

For example, a function $u \in W^{1,2}(\Omega)$ is generalized (weak) solution to the Laplace–Beltrami equation on the set $\Omega \subset \mathbb{R}^n$ if it satisfies the relation

$$\int_\Omega \nabla u \nabla v \, dx = 0$$

for all nonnegative functions $v \in C_0^1(\Omega)$.

But in the works [2] (pp. 237–240), [3] (pp. 34–40) we find other approximation approach to the determination of generalized solutions to elliptic equations. This approach arises first in the works of Poincaré of the late 19th century. He created the spreading method, which enables us to consider solutions to the Dirichlet problem free of any restrictions on domains, but keeping classical assumptions concerning continuity of the boundary data. This method had a strong influence on the further development of the theory of boundary-value problems for elliptic equations. Its ideas are embodied by O. Perron, L. Vallée Poussin, M. V. Keldysh, E. M. Landis, A. A. Grigor'ian and others.

We use terminology of the works [2] and [3]. We develop approximation approach to building of generalized solutions to boundary-value problems based on the spreading method for quasi-linear equation

$$Lu = g(x, u) \tag{1}$$

on noncompact Riemannian manifolds.

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