*Russian Mathematics (Iz. VUZ)* *Izvestiya VUZ. Matematika*
*Vol. 45, No. 9, pp.17–20, 2001* *UDC 517.544*

# RIEMANN BOUNDARY VALUE PROBLEM FOR HALF-PLANE WITH COEFFICIENTS EXPONENTIALLY DECREASING AT INFINITY

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*Introduction.* Let $\Pi_\pm = \{x + iy : x \in R,\ \pm y > 0\}$ be the upper and lower half-planes. The Riemann boundary value problem is stated as follows (see [1]–[4]). It is required to find the two vanishing at infinity in the upper and lower half-planes functions $F^+(z)$, $F^-(z)$ (a piecewise holomorphic function $F(z)$), whose limit values on the real axis $R$ satisfy the boundary value condition:

$$F^+(x) = G(x)F^-(x) + g(x) \ \text{ for almost all } \ x \in R, \tag{1}$$

where the given functions $G(t)$ and $g(t)$ are the coefficient and the free term of the problem, respectively.

It is assumed that

$$G \in L_\infty(R),\ g \in L_2(R), \tag{2}$$

and the coefficient of the problem decreases exponentially at infinity,

$$G(t) = O(e^{-bt}) \ \text{ for } \ t \to \infty, \quad b > 0. \tag{3}$$

In addition, we assume that on a set of a positive measure the coefficient of the problem is not equal zero identically (otherwise problem (1) is trivial).

A solution of the boundary value problem will be sought in the Hardy classes $H^2$

$$F^\pm(z) \in H^2(\Pi_\pm). \tag{4}$$

In the above statement the problem is new in comparison with cases considered earlier (see, e. g., [1]–[4]).

In this article by means of the Fourier transform of the boundary value condition (1) the Riemann problem (1)–(4) is reduced to the Cauchy problem for a function analytic in a strip. This enables us to find conditions of solvability and formulas of the Carleman type (see [5]) for solving the desired Riemann problem (1)–(4); the uniqueness theorem is proved.

**1.** *The Riemann boundary value problem for a piecewise holomorphic function.* The basic results of this article are represented by the following two theorems.

**Theorem 1.** *The Riemann boundary value problem* (1)–(4) *possesses at most one solution.*

Supported by the Russian Foundation for Basic Research, grant 99-01-00540.