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Instability of Systems with Linear Delay Reducible to Singularly Perturbed Ones

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Abstract—We investigate the stability of linear systems of linear delay differential equations in the case when one of subsystems is singular. We establish sufficient conditions for the instability of solutions to such systems. We solve the considered problem with the help of the Laplace transformation.

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1. INTRODUCTION

We consider the following system of linear delay differential equations:

$$\begin{aligned} \frac{d\widehat{x}(t)}{dt} &= \frac{1}{t}\Big(A_1\widehat{x}(t) + A_2\widehat{x}(\mu t) + B_1\widehat{y}(t) + B_2\widehat{y}(\mu t)\Big), \\ \frac{d\widehat{y}(t)}{dt} &= A_3\widehat{x}(t) + A_4\widehat{x}(\mu t) + B_3\widehat{y}(t) + B_4\widehat{y}(\mu t), \end{aligned} \tag{1}$$

where $\widehat{x} = \widehat{x}(t)$ and $\widehat{y} = \widehat{y}(t)$ are m-dimensional vector functions of time $t \geq t_0 > 0$, $\mu = \text{const}$, $0<\mu<1$, A_j, B_j $(j = 1, 2, 3, 4)$ are constant $m \times m$ matrices; here matrices A_2, A_4, B_2, and B_4 are assumed to be nonzero.

The goal of this paper is establishing sufficient conditions for coefficients of system (1) which make its solution instable. Sufficient conditions for the asymptotic stability were obtained earlier in [1]. Note that one can find more simple systems, for example, in [2].

In order to adduce the necessary definitions and to state the main result, let us reduce system (1) (replacing the argument $\tau = \ln(t/t_0)$) to the following constant delay system

$$\frac{dx(\tau)}{d\tau} = A_1x(\tau) + A_2x(\tau-\sigma) + B_1y(\tau) + B_2y(\tau-\sigma), \tag{2}$$

$$\frac{dy(\tau)}{d\tau} = t_0e^{\tau}\left(A_3x(\tau) + A_4x(\tau-\sigma) + B_3y(\tau) + B_4y(\tau-\sigma)\right), \tag{3}$$

where $\tau \geq 0$ and $\sigma = -\ln(\mu)$.

For the initial time moment $\tau = 0$ a solution to this system $\{x(\tau, \phi_1(\eta)),\ y(\tau, \phi_2(\eta))\}$ is uniquely defined by the initial vector function $\phi(\eta) = \big\{\phi_1(\eta), \phi_2(\eta)\big\}^\top$, $\eta \in [-\sigma, 0]$ (the sign $^\top$ means the vector transposition). Let $\phi(\eta)$ differ from the identical zero function. Without loss of generality, we assume that $\phi_1(\eta)$ and $\phi_2(\eta)$ are continuously differentiable vector functions, because as $\tau \to +\infty$ the well-known "solution smoothing" process takes place (see [2], P. 63).

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