ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2013, Vol. 57, No. 3, pp. 51–55. © Allerton Press, Inc., 2013.
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Reconstruction of a Pure State from Incomplete Information on Its Optical Tomogram

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Received August 7, 2012

Abstract—We consider the problem of reconstructing a state (i.e., a positive unit-trace operator) from incomplete information on its optical tomogram. For the case, when a (pure) state is determined by a function representing a linear combination of N ground and excited states of a quantum oscillator, we propose a technique for reconstructing this state from N values of its tomogram. For $N = 3$ we find an exact solution to the problem under consideration.

DOI: 10.3103/S1066369X13030079

Keywords and phrases: *state, optical tomogram, eigenfunctions of integral operator.*

1. INTRODUCTION

A positive operator with unit trace $\widehat{\rho}$ in a Hilbert space H is called a *quantum state*. In this paper we are dealing with the space $H = L^2(\mathbb{R})$, where the standard coordinate operator $\widehat{q}$ and the momentum operator $\widehat{p}$ act in their definition domain as follows:

$$(\widehat{q}\psi)(x) = x\psi(x), \ \ (\widehat{p}\psi)(x) = -i\frac{d\psi(x)}{dx}.$$

The authors of the paper [1] have proposed to associate every state $\widehat{\rho}$ with a family of probability distributions $\omega(x, \mu, \nu)$ by the formula

$$\omega(x, \mu, \nu) = \mathrm{Tr}(\widehat{\rho}\delta(x - \mu\widehat{q} - \nu\widehat{p})), \tag{1}$$

where μ and ν are real parameters. Family (1) is called a *symplectic quantum tomogram*; it satisfies the inversion formula

$$\widehat{\rho} = \frac{1}{2\pi}\int_{\mathbb{R}^3} e^{ix}e^{-i\mu\widehat{q}-i\nu\widehat{p}}\omega(x, \mu, \nu)dx\,d\mu\,d\nu.$$

The function $\omega(x, \mu, \nu)$ is homogeneous, namely,

$$\omega(\lambda x, \lambda\mu, \lambda\nu) = \frac{1}{|\lambda|}\omega(x, \mu, \nu).$$

Hence, it is completely determined by its values on the circle $\mu^2 + \nu^2 = 1$. We put

$$W(x, \theta) = \omega(x, \cos(\theta), \sin(\theta)), \ \ \theta \in [0, 2\pi).$$

The value $W(x, \theta)$ is called an *optical tomogram*.

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