ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2007, Vol. 51, No. 6, pp. 61–74. © Allerton Press, Inc., 2007.
Original Russian Text © B.V. Simonov, 2007, published in Izvestiya Vysshikh Uchebnykh Zavedenii. Matematika, 2007, No. 6, pp. 64–76.

Trigonometric Series in the Orlicz–Lorentz Spaces

V. B. Simonov[*]

Volgograd State Technical University, pr. Lenina 28, Volgograd, 400131 Russia

Received December 23, 2005

DOI: 10.3103/S1066369X07060084

1. INTRODUCTION

Let Φ be the set of all nonnegative on $[0,+\infty)$ almost increasing [1] functions (i. e., $\varphi(x_1) \le C_1\varphi(x_2)$ for any $0 < x_1 < x_2 < +\infty$, where $C_1 > 0$ is independent of x_1, x_2), satisfying the Δ_2-condition (i. e., $\varphi(2x) \le C_2\varphi(x)$ for any $0 < x < +\infty$, where $C_2 > 0$ is independent of x); let W be the set of all measurable, nonnegative almost everywhere on $(0,2\pi)$ functions w. If $\varphi \in \Phi$, $w \in W$, then the Orlicz–Lorentz space $\Lambda(\varphi,w)$ is the set of all 2π-periodic measurable functions $f(x)$ such that $\|f\|_{\varphi,w} = \int\limits_0^{2\pi} w(t)\varphi(f^*(t))dt < +\infty$, where $f^*(t)$ is a nonincreasing on $[0,2\pi]$ function, equimeasurable with $|f|$.

Consider the trigonometric series in the forms

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx \tag{1}$$

and

$$\sum_{n=1}^{\infty} a_n \sin nx. \tag{2}$$

The following theorem is well-known.

The Hardy–Littlewood theorem ([2], P. 657). *Assume that $a_n \to 0$ for $n \to \infty$ and $a_n \ge a_{n+1}$ for all n, where a_n are the coefficients of series* (1), (2), *and $f(x)$, $g(x)$ are their sums. Then for $p \in (1,\infty)$ the following inequalities are true*:

$$C_1\Big(\sum_{n=0}^{\infty} a_n^p (n+1)^{p-2}\Big)^{1/p} \le \Big(\int_0^{2\pi} |f(x)|^p dx\Big)^{1/p} \le C_2\Big(\sum_{n=0}^{\infty} a_n^p (n+1)^{p-2}\Big)^{1/p},$$

$$C_3\Big(\sum_{n=1}^{\infty} a_n^p n^{p-2}\Big)^{1/p} \le \Big(\int_0^{2\pi} |g(x)|^p dx\Big)^{1/p} \le C_4\Big(\sum_{n=1}^{\infty} a_n^p n^{p-2}\Big)^{1/p};$$

here the positive constants C_1, C_2, C_3, C_4 are independent of the sequence $\{a_n\}$.

Note that these inequalities and all those adduced below are understood in the following sense: the finiteness of the right-hand side implies the finiteness of the left-hand one. Below $C_1, C_2, \ldots$ stand for positive constants, not necessarily the same in various formulas.

The following theorem is proved in [3].

[*]E-mail: htf212@vstu.ru.